Iowa State University

ECpE Department

EE653 Power distribution system modeling, optimization and simulation

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Modeling Three-Phase Transformers

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Three-Phase Transformer Models

- Three-phase transformer banks are found in the distribution substation where the voltage is transformed from the transmission or subtransmission level to the distribution feeder level.
- In most cases, the substation transformer will be a three-phase unit perhaps with high-voltage no-load taps and, perhaps, low-voltage load tap changing (LTC).
- For a four-wire wye feeder, the most common substation transformer connection is the delta–grounded wye. A three-wire delta feeder will typically have a delta–delta transformer connection in the substation. Three-phase transformer banks out on the feeder will provide the final voltage transformation to the customer's load. A variety of transformer connections can be applied. The load can be pure three-phase or a combination of single-phase lighting load and a three-phase load such as an induction motor. In the analysis of a distribution feeder, it is important that the various three-phase transformer connections be modeled correctly.

Three-Phase Transformer Models

Unique models of three-phase transformer banks applicable to radial distribution feeders will be developed. Models for the following three-phase connections are included:

- Delta–grounded wye
- Ungrounded wye–delta
- Grounded wye–delta
- Open wye–open delta
- Grounded wye–grounded wye
- Delta–delta
- Open delta–open delta

Introduction

Fig.1 defines the various voltages and currents for all three-phase transformer banks connected between the source-side node ***n*** and the load-side node ***m***.

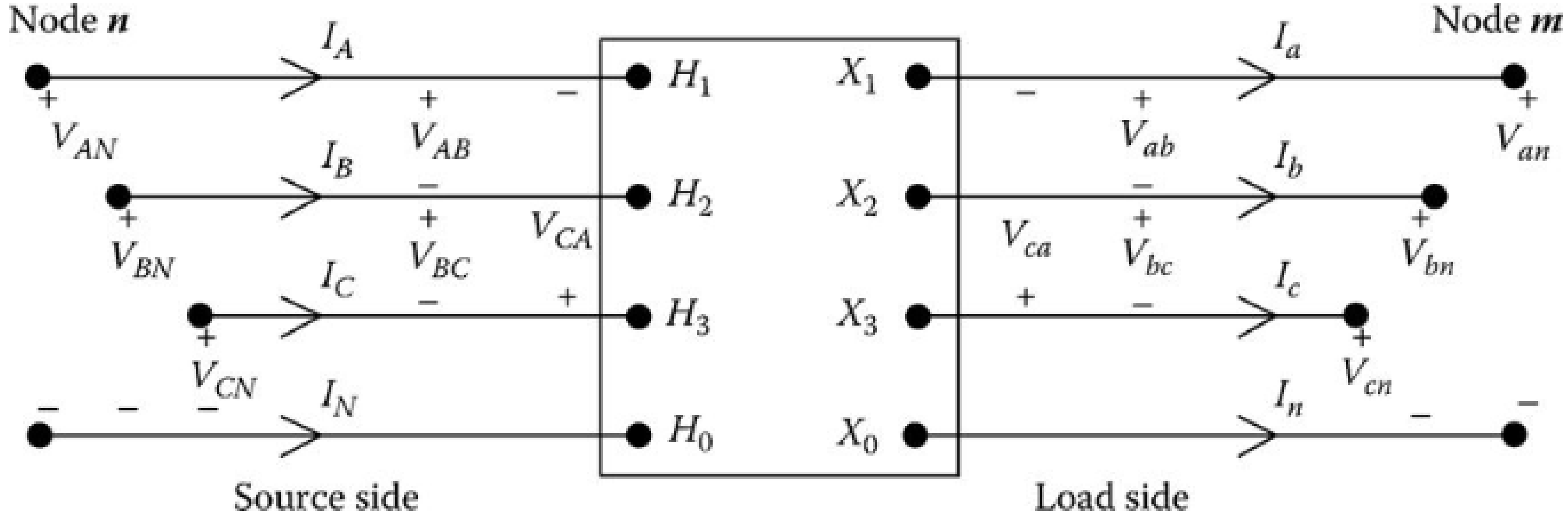


Fig.1 General three-phase transformer bank

Introduction

In Fig.1, the models can represent a step-down (source side to load side) or a step-up (source side to load side) transformer bank. The notation is such that the capital letters *A, B, C*, and *N* will always refer to the *source* side (node ***n***) of the bank and the lower case letters *a, b, c*, and *n* will always refer to the **load** side (node ***m***) of the bank. It is assumed that all variations of the wye–delta connections are connected in the "American Standard Thirty Degree" connection. The described phase notation and the standard phase shifts for positive sequence voltages and currents are

Step-down connection

$$V_{AB} \; leads \; V_{ab} \text{ by } 30^{\circ} \quad (1)$$

$$I_{A} \; leads \; I_{a} \text{ by } 30^{\circ} \quad (2)$$

Step-up connection

$$V_{ab} \; leads \; V_{AB} \text{ by } 30^{\circ} \quad (3)$$

$$I_{a} \; leads \; I_{A} \text{ by } 30^{\circ} \quad (4)$$

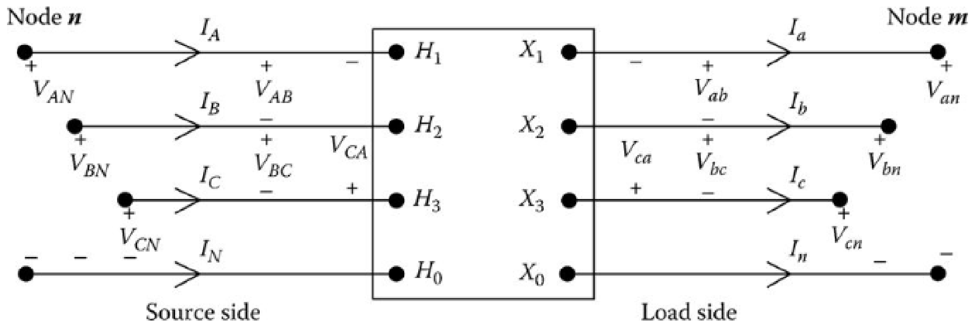


Fig.1 General three-phase transformer bank

Generalized Matrices

The models to be used in power-flow and short-circuit studies are generalized for the connections in the same form as have been developed for line segments and voltage regulators. In the “forward sweep” of the “ladder” iterative technique, the voltages at node ***m*** are defined as a function of the voltages at node ***n*** and the currents at node ***m***. The required equation is

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \quad (5)$$

In the “backward sweep” of the ladder technique, the matrix equations for computing the voltages and currents at node ***n*** as a function of the voltages and currents at node ***m*** are given by

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}] \quad (7)$$

Generalized Matrices

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \quad (5)$$

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}] \quad (7)$$

In Equations (5) through (7), the matrices $[VLN_{ABC}]$ and $[VLN_{abc}]$ represent the line-to-neutral voltages for an ungrounded wye connection or the line-to-ground voltages for a grounded wye connection. For a delta connection, the voltage matrices represent "equivalent" line-to-neutral voltages. The current matrices represent the line currents regardless of the transformer winding connection.

In the modified ladder technique, Equation (5) is used to compute new node voltages downstream from the source using the most recent line currents. In the backward sweep, only Equation (7) is used to compute the source-side line currents using the newly computed load-side line currents.

Transformer model

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \qquad (8.5)$$

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \qquad (8.6)$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}] \qquad (8.7)$$

Transformer model

	$\triangle$ − Grounded Y Step-down	$\triangle$ − Grounded Y Step-up	Ungrounded Y − $\triangle$ Step-down	Grounded Y − Grounded Y
$[a_t]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2 & 1\\1 & 0 & 2\\2 & 1 & 0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2 & 1 & 0\\0 & 2 & 1\\1 & 0 & 2\end{bmatrix}$	$n_t\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}n_t & 0 & 0\\0 & n_t & 0\\0 & 0 & n_t\end{bmatrix}$
$[b_t]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2Z_{t_b} & Z_{t_c}\\Z_{t_a} & 0 & 2Z_{t_c}\\2Z_{t_a} & Z_{t_b} & 0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2Z_{t_a} & Z_{t_b} & 0\\0 & 2Z_{t_b} & Z_{t_c}\\Z_{t_a} & 0 & 2Z_{t_c}\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}Z_{t_{ab}} & -Z_{t_{ab}} & 0\\Z_{t_{bc}} & 2Z_{t_{bc}} & 0\\-2Z_{t_{ca}} & Z_{t_{ca}} & 0\end{bmatrix}$	$\begin{bmatrix}n_tZ_{t_a} & 0 & 0\\0 & n_tZ_{t_b} & 0\\0 & 0 & n_tZ_{t_c}\end{bmatrix}$
$[c_t]$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$
$[d_t]$	$\frac{1}{n_t}\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}1 & -1 & 0\\1 & 2 & 0\\-2 & -1 & 0\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
$[A_t]$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\-1 & 1 & 0\\0 & -1 & 1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}2 & 1 & 0\\0 & 2 & 1\\1 & 0 & 2\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
$[B_t]$	$\begin{bmatrix}Z_{t_a} & 0 & 0\\0 & Z_{t_b} & 0\\0 & 0 & Z_{t_c}\end{bmatrix}$	$[Z_{t_{abc}}]$	$\frac{1}{9}\begin{bmatrix}2Z_{t_{ab}} + Z_{t_{bc}} & 2Z_{t_{bc}} - 2Z_{t_{ab}} & 0\\2Z_{t_{bc}} - 2Z_{t_{ca}} & 4Z_{t_{bc}} - Z_{t_{ca}} & 0\\Z_{t_{ab}} - 4Z_{t_{ca}} & -Z_{t_{ab}} - 2Z_{t_{ca}} & 0\end{bmatrix}$	$[Z_{t_{abc}}]$
n_t	$\frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$	$\frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}}$	$\frac{VLN_{rated\ primary}}{VLL_{rated\ secondary}}$	$\frac{VLN_{rated\ primary}}{VLN_{rated\ secondary}}$

Delta-Grounded Wye Step-Down Connection

- The delta–grounded wye step-down connection is a popular connection that is typically used in a distribution substation serving a four-wire wye feeder system. Another application of the connection is to provide service to a load that is primary single phase. Because of the wye connection, three single-phase circuits are available thereby making it possible to balance the single-phase loading on the transformer bank.
- Three single-phase transformers can be connected delta–grounded wye in a "standard thirty degree step-down connection" as shown in Fig.2.

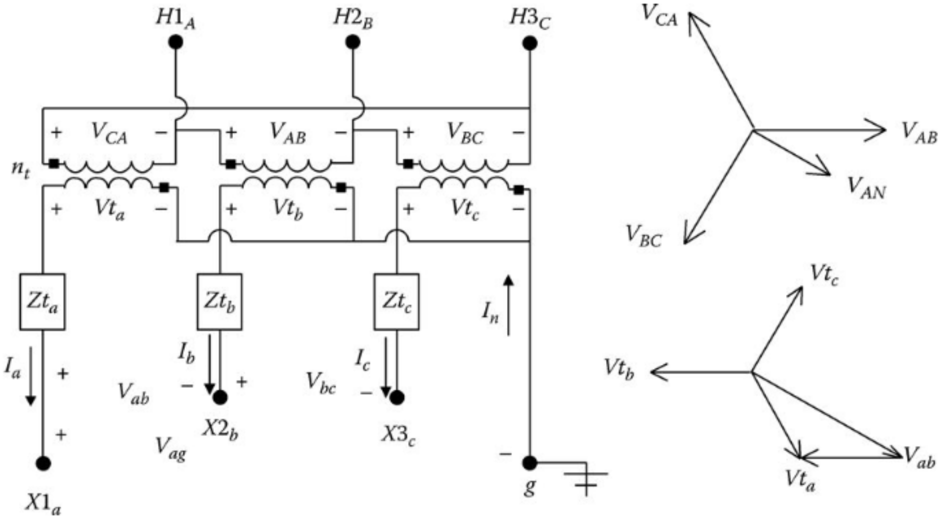


Fig.2 Standard delta-grounded wye connection with voltage

Voltages

The positive sequence phasor diagrams of the voltages in Fig.2 show the relationships between the various positive sequence voltages. Care must be taken to observe the polarity marks on the individual transformer windings. In order to simplify the notation, it is necessary to label the "ideal" voltages with voltage polarity markings as shown in Fig.2. Observing the polarity markings of the transformer windings, the voltage Vt_a will be 180° out of phase with the voltage V_{CA} and the voltage Vt_b will be 180° out of phase with the voltage V_{AB}. Kirchhoff's voltage law (KVL) gives the line-to-line voltage between phases a and b as

$$V_{ab} = Vt_a - Vt_b \qquad (8)$$

The phasors of the positive sequence voltages of Equation (8) are shown in Fig.2.

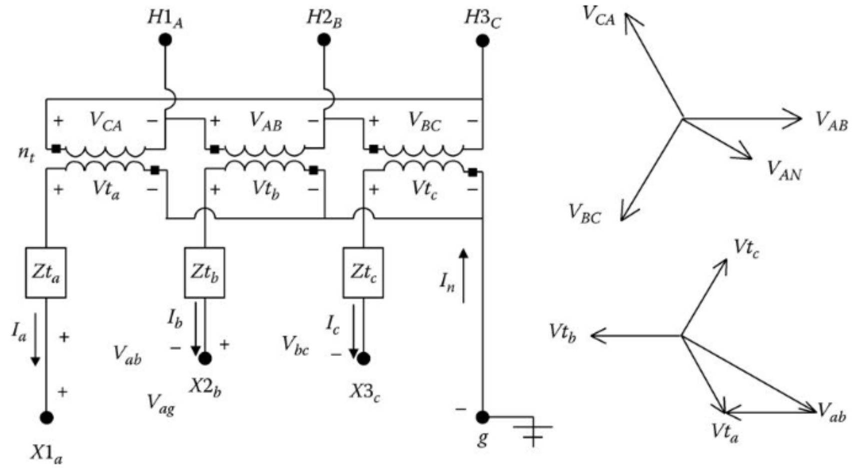


Fig.2 Standard delta-grounded wye connection with voltage

Voltages

The magnitude changes between the voltages can be defined in terms of the actual winding turns ratio (n_t). With reference to Fig.2, these ratios are defined as follows:

$$n_t = \frac{VLL_{rated\ primary}}{VLN_{rated\ secondary}} \tag{9}$$

With reference to Fig.2, the line-to-line voltages on the primary side of the transformer connection as a function of the ideal secondary side voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix}; [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{10}$$

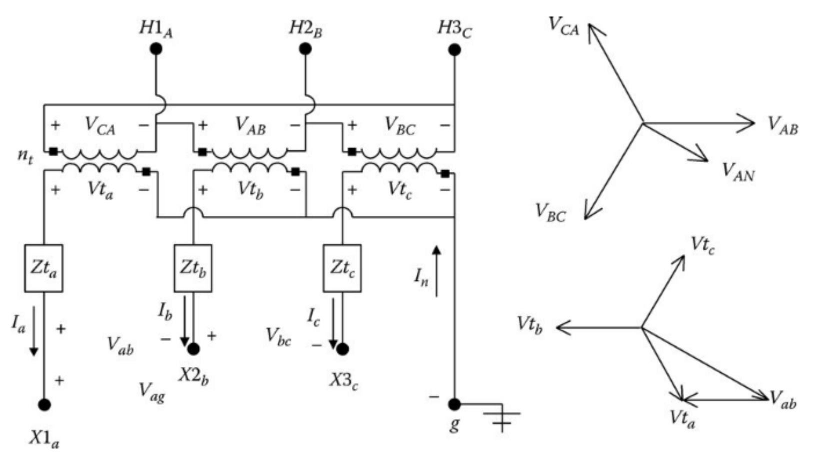


Fig.2 Standard delta-grounded wye connection with voltage

where

$$[AV] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Voltages

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix}; [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \quad (10)$$

Equation (10) gives the primary line-to-line voltages at node ***n*** as a function of the ideal secondary voltages. However, what is needed is a relationship between "equivalent" line-to-neutral voltages at node ***n*** and the ideal secondary voltages. The question is how is the equivalent line-to-neutral voltages determined knowing the line-to-line voltages? One approach is to apply the theory of symmetrical components.

The known line-to-line voltages are transformed to their sequence voltages by

$$[VLL_{012}] = [A_s]^{-1} \cdot [VLL_{ABC}] \quad (11)$$

where

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix}$$

$$a_s = 1.0\angle 120$$

Voltages

By definition, the zero sequence line-to-line voltage is always zero. The relationship between the positive and negative sequence line-to-neutral and line-to-line voltages is known. These relationships in matrix form are given by

$$\begin{bmatrix} VLN_0 \\ VLN_1 \\ VLN_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \cdot \begin{bmatrix} VLL_0 \\ VLL_1 \\ VLL_2 \end{bmatrix}; [VLN_{012}] = [T] \cdot [VLL_{012}] \quad (13)$$

where

$$t = \frac{1}{\sqrt{3}} \angle 30$$

Since the zero sequence line-to-line voltage is zero, the (1,1) term of the matrix $[T]$ can be any value. For the purposes here, the (1,1) term is chosen to have a value of 1.0. Knowing the sequence line-to-neutral voltages, the equivalent line-to-neutral voltages can be determined.

Voltages

$$\begin{bmatrix} VLN_0 \\ VLN_1 \\ VLN_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \cdot \begin{bmatrix} VLL_0 \\ VLL_1 \\ VLL_2 \end{bmatrix}; \ [VLN_{012}] = [T] \cdot [VLL_{012}] \tag{13}$$

The equivalent line-to-neutral voltages as a function of the sequence line-to-neutral voltages are

$$[VLN_{ABC}] = [A_s] \cdot [VLN_{012}] \tag{14}$$

Substitute Equation (13) into Equation (14):

$$[VLN_{ABC}] = [A_s] \cdot [T] \cdot [VLL_{012}] \tag{15}$$

$$[VLL_{012}] = [A_s]^{-1} \cdot [VLL_{ABC}] \tag{11}$$

Substitute Equation (11) into Equation (15):

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \tag{16}$$

where

$$[W] = [A_s] \cdot [T] \cdot [A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{17}$$

Voltages

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \tag{16}$$

$$[W] = [A_s] \cdot [T] \cdot [A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{17}$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{10}$$

Equation (17) provides a method of computing equivalent line-to-neutral voltages from a knowledge of the line-to-line voltages. This is an important relationship that will be used in a variety of ways as other three-phase transformer connections are studied.
To continue on, Equation (16) can be substituted into Equation (10):

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] = [W] \cdot [AV] \cdot [Vt_{abc}] = [a_t] \cdot [Vt_{abc}] \tag{18}$$

where

$$[a_t] = [W] \cdot [AV] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \tag{19}$$

Voltages

$$[a_t] = [W] \cdot [AV] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \quad (19)$$

Equation (19) defines the generalized $[a_t]$ matrix for the delta–grounded wye step-down connection.

The ideal secondary voltages as a function of the secondary line-to-ground voltages and the secondary line currents are

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \quad (20)$$

where

$$[Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \quad (21)$$

Note in Equation (21) there is no restriction that the impedances of the three transformers be equal.

Voltages

$$[VLN_{ABC}] = [W] \cdot [VLL] = [W] \cdot [AV] \cdot [Vt_{abc}] = [a_t] \cdot [Vt_{abc}] \quad (18)$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \quad (20)$$

Substitute Equation (20) into Equation (18):

$$[VLN_{ABC}] = [a_t] \cdot ([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}])$$

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \quad (22)$$

where

$$[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \\ 2 \cdot Zt_a & Zt_b & 0 \end{bmatrix} \quad (23)$$

Voltages

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix}; \quad [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad (10)$$

The generalized matrices $[a_t]$ and $[b_t]$ have now been defined. The derivation of the generalized matrices $[A_t]$ and $[B_t]$ begins with solving Equation (10) for the ideal secondary voltages:

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLL_{ABC}] \qquad (24)$$

The line-to-line voltages as a function of the equivalent line-to-neutral voltages are

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] \qquad (25)$$

where

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad (26)$$

Voltages

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLL_{ABC}] \quad (24)$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] \quad (25)$$

Substitute Equation (25) into Equation (24):

$$[Vt_{abc}] = [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] = [A_t] \cdot [VLN_{ABC}] \quad (27)$$

where

$$[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (28)$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \quad (20)$$

Substitute Equation (20) into Equation (27):

$$[VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] = [A_t] \cdot [VLN_{ABC}] \quad (29)$$

Voltages

$$[VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] = [A_t] \cdot [VLN_{ABC}] \quad (29)$$

Rearrange Equation (29)

$$[VLG_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \quad (30)$$

where

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \quad (31)$$

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \quad (22)$$

Equation (22) is referred to as the "backward sweep voltage equation" and Equations (30) is referred to as the "forward sweep voltage equation." Equations (22) and (30) apply only for the step-down delta–grounded wye transformer. Note that these equations are in exactly the same form as those derived in earlier chapters for line segments and step-voltage regulators.

Current

The thirty degree connection specifies that the positive sequence current entering the H_1 terminal will lead the positive sequence current leaving the X_1 terminal by 30°. Fig.3 shows the same connection as Fig.2 but with the currents instead of the voltages displayed.

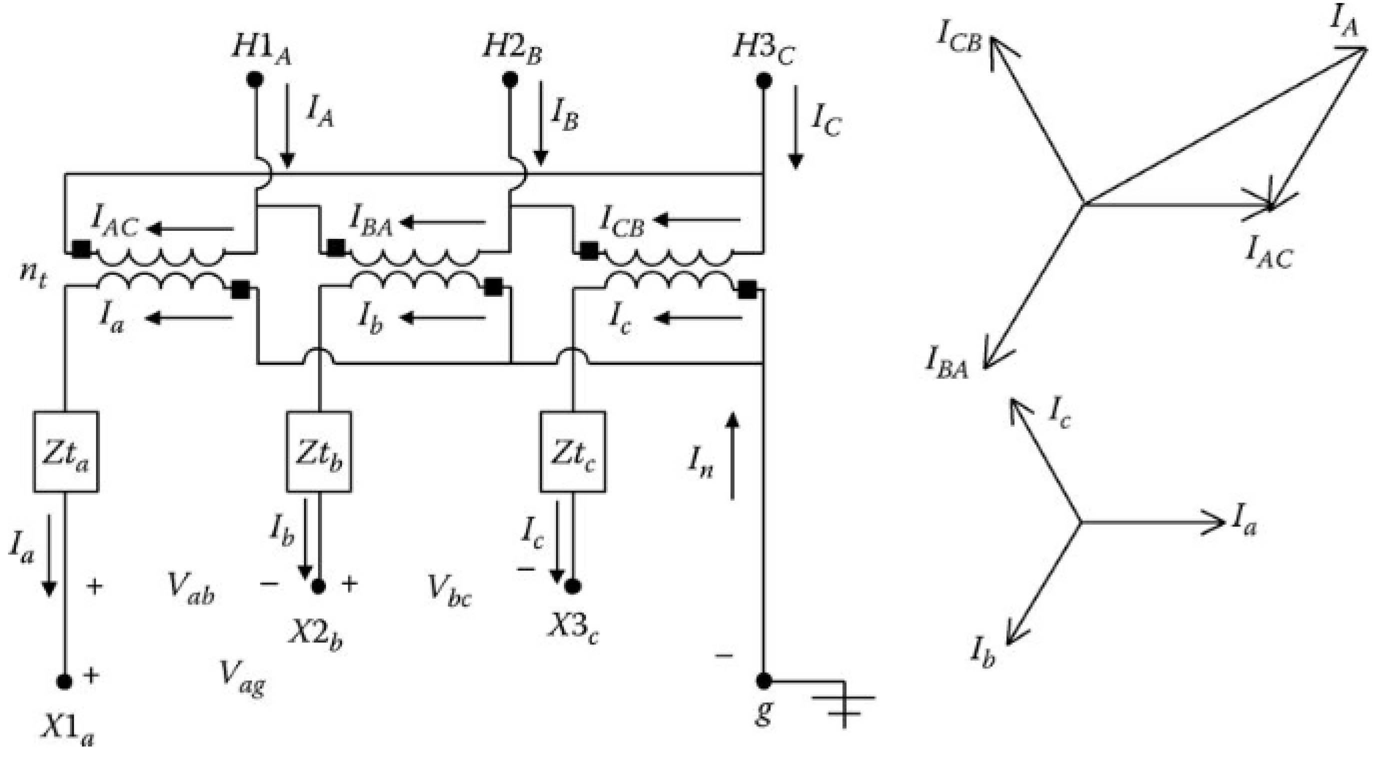


Fig.3 Delta-grounded wye connection with current

Current

As with the voltages, the polarity marks on the transformer windings must be observed for the currents. For example, in Fig.3, the current I_a is entering the polarity mark on the low-voltage winding so the current I_{AC} flowing out of the polarity mark on the high-voltage winding will be in phase with I_a. This relationship is shown in the phasor diagrams for positive sequence currents in Fig.3.

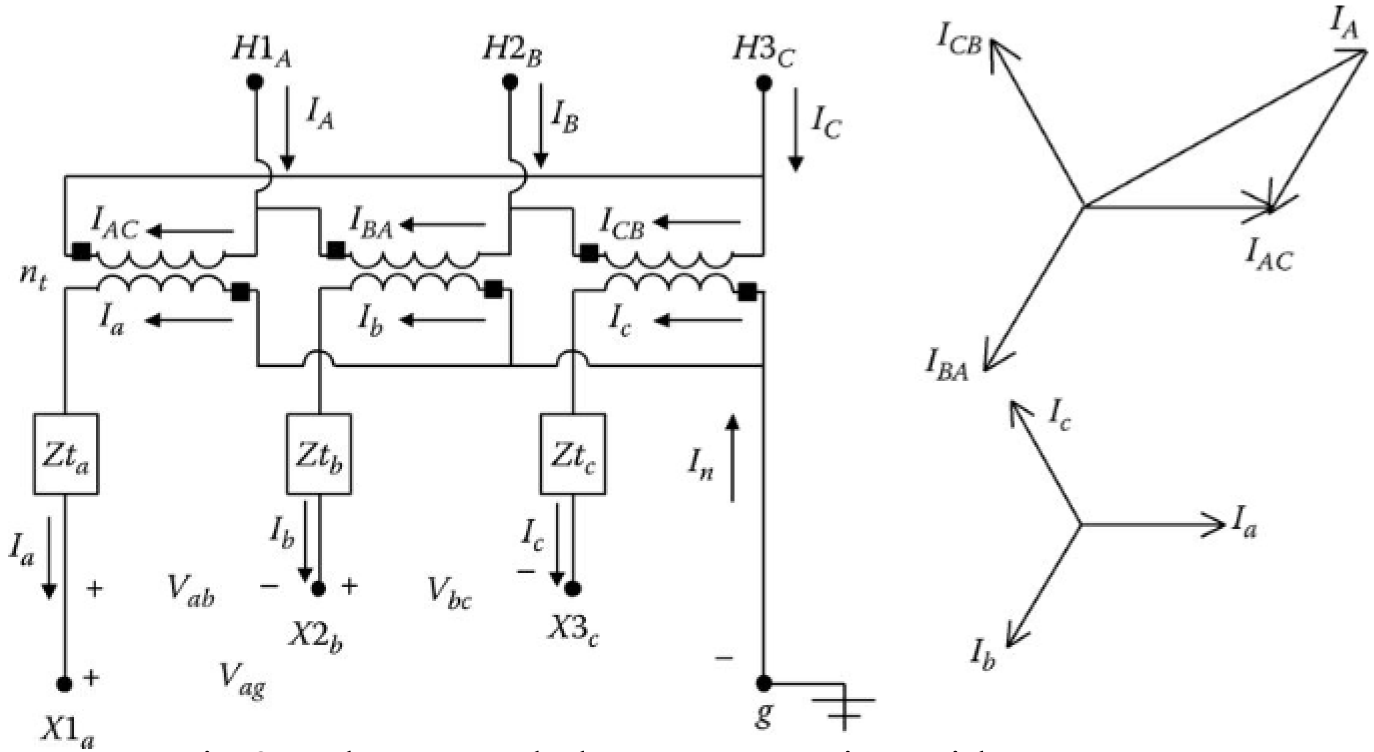


Fig.3 Delta-grounded wye connection with current

Current

The line currents can be determined as a function of the delta currents by applying Kirchhoff's current law (KCL):

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{Ac} \\ I_{BA} \\ I_{CB} \end{bmatrix} \tag{32}$$

In condensed form, Equation (32) is

$$[I_{ABC}] = [D] \cdot [ID_{ABC}] \tag{33}$$

where

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

The matrix equation relating the delta primary currents to the secondary line currents is given by

$$\begin{bmatrix} I_{Ac} \\ I_{BA} \\ I_{CB} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{34}$$

$$[ID_{ABC}] = [AI] \cdot [I_{abc}] \qquad [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{35}$$

Current

$$[I_{ABC}] = [D] \cdot [ID_{ABC}]$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (33)$$

$$[AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (35)$$

Substitute Equation (35) into Equation (33):

$$[I_{ABC}] = [D] \cdot [AI] \cdot [I_{abc}] = [c_t] \cdot [VLG_{abc}] + [d_t] \cdot [I_{abc}] \quad (36)$$

where

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (37)$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (38)$$

Current

$$[I_{ABC}] = [D] \cdot [AI] \cdot [I_{abc}] = [c_t] \cdot [VLG_{abc}] + [d_t] \cdot [I_{abc}] \qquad (36)$$

Equation (36) (referred to as the "backward sweep current equations") provides a direct method of computing the phase line currents at node $\boldsymbol{n}$ knowing the phase line currents at node $\boldsymbol{m}$. Again, this equation is in the same form as that previously derived for three-phase line segments and three-phase step-voltage regulators.

The equations derived in this section are for the step-down connection. Section 8.4 summarizes the matrices for the delta–grounded wye step-up connection.

Example 1

In the example system of Figure 4, an unbalanced constant impedance load is being served at the end of a 1 mile section of a three-phase line. The 1 mile long line is being fed from a substation transformer rated 5000 kVA, 115 kV delta–12.47 kV grounded wye with a per-unit impedance of 0.085 ∠ 85.

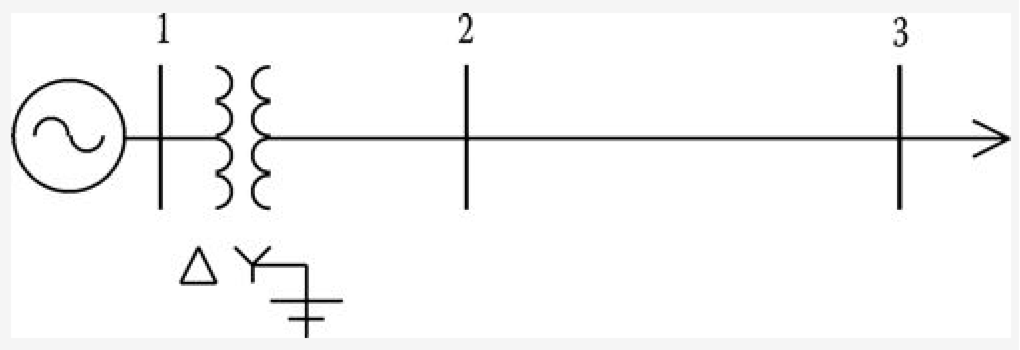


Fig.4 Example system

The phase conductors of the line are 336,400 26/7 ACSR with a neutral conductor 4/0 ACSR. The configuration and computation of the phase impedance matrix are given in Example 4.1. From that example, the phase impedance matrix was computed to be

$$[Zline_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/mile$$

Example 1

The general matrices for the line are

$$[A_{line}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [B_{line}] = [Zline_{abc}] \qquad [d] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformer impedance needs to be converted to per unit referenced to the low-voltage side of the transformer. The base impedance is

$$Z_{base} = \frac{12.47^2 \cdot 1000}{5000} = 31.1\Omega$$

The transformer impedance referenced to the low-voltage side is

$$Z_t = (0.085\angle 85) \cdot 31.1 = 0.2304 + j2.6335\ \Omega$$

The transformer phase impedance matrix is

$$[Zt_{abc}] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix} \Omega$$

Example 1

The unbalanced constant impedance load is connected in grounded wye. The load impedance matrix is specified to be

$$[Zload_{abc}] = \begin{bmatrix} 12+j6 & 0 & 0 \\ 0 & 13+j4 & 0 \\ 0 & 0 & 14+j5 \end{bmatrix} \Omega$$

The unbalanced line-to-line voltages at node 1 serving the substation transformer are given as

$$[VLL_{ABC}] = \begin{bmatrix} 115{,}000\angle 0 \\ 116{,}500\angle -115.5 \\ 123{,}538\angle 121.7 \end{bmatrix} V$$

A. Determine the generalized matrices for the transformer. The "transformer turn's" ratio is

$$n_t = \frac{KVLL_{high}}{KVLN_{low}} = \frac{115}{12.47/\sqrt{3}} = 15.9732$$

Example 1

From Equation (19)

$$[a_t] = [W] \cdot [AV] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \quad (19)$$

$$[a_t] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -10.6488 & -5.3244 \\ -5.3244 & 0 & -10.6488 \\ -10.6488 & -5.3244 & 0 \end{bmatrix}$$

From Equation (23)

$$[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \\ 2 \cdot Zt_a & Zt_b & 0 \end{bmatrix} \quad (23)$$

$$[b_t] = \begin{bmatrix} 0 & -2.4535 - j28.0432 & -1.2267 - j14.0216 \\ -1.2267 - j14.0216 & 0 & -2.4535 - j28.0432 \\ -2.4535 - j28.0432 & -1.2267 - j14.0216 & 0 \end{bmatrix}$$

Example 1

From Equation (37)

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{37}$$

$$[d_t] = \begin{bmatrix} 0.0626 & -0.0626 & 0 \\ 0 & 0.0626 & -0.0626 \\ -0.0626 & 0 & 0.0626 \end{bmatrix}$$

From Equation (28)

$$[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{28}$$

$$[A_t] = \begin{bmatrix} 0.0626 & 0 & -0.0626 \\ -0.0626 & 0.0626 & 0 \\ 0 & -0.0626 & 0.0626 \end{bmatrix}$$

Example 1

From Equation (31)

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_a & 0 \\ 0 & 0 & Zt_a \end{bmatrix} \qquad (31)$$

$$[B_t] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix}$$

Example 1

B. Given the line-to-line voltages at node 1, determine the "ideal" transformer voltages. From Equation (27),

$$[AV] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -15.9732 & 0 \\ 0 & 0 & -15.9732 \\ -15.9732 & 0 & 0 \end{bmatrix}$$

$$[Vt_{abc}] = [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 7734.1\angle -58.3 \\ 7199.6\angle 180 \\ 7293.5\angle 64.5 \end{bmatrix} V$$

Example 1

C. Determine the load currents.

Since the load is modeled as constant impedances, the system is linear and the analysis can combine all of the impedances (transformer, line, and load) to an equivalent impedance matrix. KVL gives

$$[Vt_{abc}] = ([Zt_{abc}] + [Zline_{abc}] + [Zload_{abc}]) \cdot [I_{abc}] = [Zeq_{abc}] \cdot [I_{abc}]$$

$$[Zeq_{abc}] = \begin{bmatrix} 13.0971 + j10.6751 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 14.1141 + j8.6187 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 15.1045 + j9.6507 \end{bmatrix} \Omega$$

The line current can now be computed:

$$[I_{abc}] = [Zeq_{abc}]^{-1} \cdot [Vt_{abc}] = \begin{bmatrix} 471.7\angle -95.1 \\ 456.7\angle 149.9 \\ 427.3\angle 33.5 \end{bmatrix} A$$

Example 1

D. Determine the line-to-ground voltages at the load in volts and on a 120 V base:

$$[Vload_{abc}] = [Zload_{abc}] \cdot [I_{abc}] = \begin{bmatrix} 6328.1\angle-68.6 \\ 6212.2\angle167.0 \\ 6352.6\angle53.1 \end{bmatrix} V$$

The load voltages on a 120 V base are

$$[Vload_{120}] = \begin{bmatrix} 105.5 \\ 103.5 \\ 105.9 \end{bmatrix}$$

The line-to-ground voltage at node 2 are

$$[VLG_{abc}] = [a_{line}] \cdot [Vload_{abc}] + [b_{line}] \cdot [I_{abc}] = \begin{bmatrix} 6965.4\angle-66.0 \\ 6580.6\angle171.4 \\ 6691.4\angle56.7 \end{bmatrix} V$$

Example 1

E. Using the backward sweep voltage equation, determine the equivalent line-to-neutral voltages and the line-to-line voltages at node 1:

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] = \begin{bmatrix} 69{,}443\angle -30.3 \\ 65{,}263\angle -147.5 \\ 70{,}272\angle 94.0 \end{bmatrix} V$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 115{,}000\angle 0 \\ 116{,}500\angle -115.5 \\ 123{,}538\angle 121.7 \end{bmatrix} V$$

It is always comforting to be able to work back and compute what was initially given. In this case, the line-to-line voltages at node 1 have been computed and the same values result that were given at the start of the problem.

Example 1

F. Use the forward sweep voltage equation to verify that the line-to-ground voltages at node 2 can be computed knowing the equivalent line-to-neutral voltages at node 1 and the currents leaving node 2:

$$[VLG_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] = \begin{bmatrix} 6965.4\angle -66.0 \\ 6580.6\angle 171.4 \\ 6691.4\angle 56.7 \end{bmatrix} V$$

These are the same values of the line-to-ground voltages at node 2 that were determined working from the load toward the source.

Example 8.1 has demonstrated the application of the forward and backward sweep equations. The example also provides verification that the same voltages and currents result working from the load toward the source or from the source toward the load.

In Example 8.2, the system of Example 8.1 is used only this time the source voltages at node 1 are specified and the three-phase load is specified as constant *PQ*. Because this makes the system nonlinear, the ladder iterative technique must be used to solve for the system voltages and currents.

Example 2

Use the system of Example 8.1. The source voltages at node 1 are

$$[VLL_{ABC}] = \begin{bmatrix} 115{,}000\angle 0 \\ 115{,}000\angle -120 \\ 115{,}000\angle 120 \end{bmatrix} V$$

The wye-connected loads are

$$[kVA] = \begin{bmatrix} 1700 \\ 1200 \\ 1500 \end{bmatrix} \qquad [PF] = \begin{bmatrix} 0.90 \\ 0.85 \\ 0.95 \end{bmatrix}$$

The complex powers of the loads are computed to be

$$SL_i = kVA_i \cdot e^{j\cdot \text{acos}(PF_i)} = \begin{bmatrix} 1530 + j741.0 \\ 1020 + j632.1 \\ 1425 + j468.4 \end{bmatrix} kW + jkvar$$

Example 2

The ladder iterative technique must be used to analyze the system. A simple Mathcad® program is shown in Fig.5.

$$\left|VLL_{ABC_i}\right| = \begin{pmatrix} 115000 \\ 115000 \\ 115000 \end{pmatrix} \qquad \frac{\arg\left(VLL_{ABC_i}\right)}{deg} = \begin{pmatrix} 0 \\ -120 \\ 120 \end{pmatrix} \qquad VM = 7199.5579$$

$$Start := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad Tol := .000001 \qquad VM := \frac{kVLL_{sec} \cdot 1000}{\sqrt{3}}$$

$$
\begin{aligned}
X := \; & I_{abc} \leftarrow Start \\
& Iload_{abc} \leftarrow Start \\
& V_{old} \leftarrow Start \\
& ELN_{ABC} \leftarrow W \cdot ELL_{ABC} \\
& \text{for } n \in 1..200 \\
& \quad V2LN_{abc} \leftarrow A_t \cdot ELN_{ABC} - B_t \cdot I_{abc} \\
& \quad V3LN_{abc} \leftarrow A_{line} \cdot V2LN_{abc} - B_{line} \cdot Iload_{abc} \\
& \quad \text{for } j \in 1..3 \\
& \quad\quad Iload_{abc_j} \leftarrow \overline{\left(\frac{SL_j \cdot 1000}{V3LN_{abc_j}}\right)} \\
& \quad \text{for } k \in 1..3 \\
& \quad\quad Error_k \leftarrow \frac{\left|V3LN_{abc_k} - V_{old_k}\right|}{VM} \\
& \quad Error_{max} \leftarrow \max(Error) \\
& \quad \text{break if } Error_{max} < Tol \\
& \quad V_{old} \leftarrow V3LN_{abc} \\
& \quad I_{abc} \leftarrow d_{line} \cdot Iload_{abc} \\
& \quad I_{ABC} \leftarrow d_t \cdot I_{abc} \\
& Out_1 \leftarrow V3LN_{abc} \\
& Out_2 \leftarrow V2LN_{abc} \\
& Out_3 \leftarrow I_{abc} \\
& Out_4 \leftarrow I_{ABC} \\
& Out_5 \leftarrow n \\
& Out
\end{aligned}
$$

Fig.5 Mathcad® program

Example 2

Note in this program that in the forward sweep the secondary transformer voltages are first computed and then those are used to compute the voltages at the loads. At the end of the routine, the newly calculated line currents are taken back to the top of the routine and used to compute the new voltages. This continues until the error in the difference between the two most recently calculated load voltages are less than the tolerance. As a last step, after conversion, the primary currents of the transformer are computed.

After nine iterations, the load voltages and currents are

$$[VLN_{load}] = \begin{bmatrix} 6490.1\angle -66.7 \\ 6772.4\angle 176.2 \\ 6699.4\angle 53.9 \end{bmatrix} \qquad [I_{abc}] = \begin{bmatrix} 261.9\angle -92.5 \\ 117.2\angle 144.4 \\ 223.9\angle 35.7 \end{bmatrix}$$

The primary currents are

$$[I_{ABC}] = \begin{bmatrix} 24.3\angle -70.0 \\ 20.5\angle -175.2 \\ 27.4\angle 63.8 \end{bmatrix}$$

Example 2

The magnitude of the load voltages on a 120 V base are

$$[Vload_{120}] = \begin{bmatrix} 108.2 \\ 112.9 \\ 111.7 \end{bmatrix}$$

Needless to say, these voltages are not acceptable. In order to correct this problem, three step-voltage regulators can be installed at the secondary terminals of the substation transformer as shown in Fig.6.

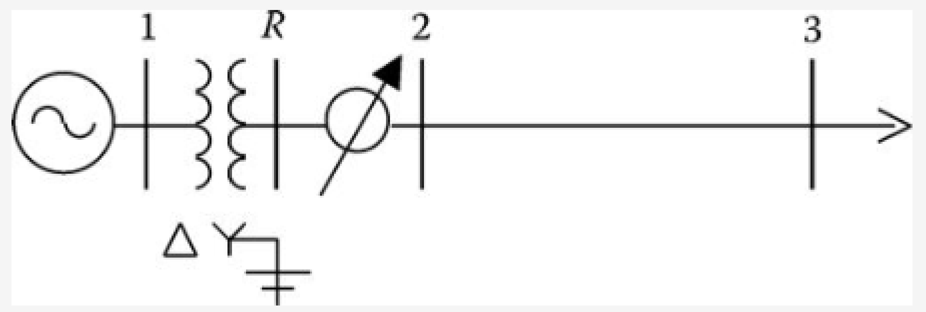


Fig.6 Voltage regulators installed

Using the method as outlined in Chapter 7, the final steps for the three regulators are

$$[Tap] = \begin{bmatrix} 14 \\ 9 \\ 10 \end{bmatrix}$$

Example 2

The regulator turns ratios are

$$aR_i = 1 - 0.00625 \cdot Tap_i = \begin{bmatrix} 0.9125 \\ 0.9438 \\ 0.9375 \end{bmatrix}$$

The regulator matrices are

$$[A_{reg}] = [d_{reg}] = \begin{bmatrix} \frac{1}{aR_1} & 0 & 0 \\ 0 & \frac{1}{aR_2} & 0 \\ 0 & 0 & \frac{1}{aR_3} \end{bmatrix} = \begin{bmatrix} 1.0959 & 0 & 0 \\ 0 & 1.0596 & 0 \\ 0 & 0 & 1.0667 \end{bmatrix}$$

$$[B_{reg}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Example 2

At the start of the Mathcad routine, the following equation is added:

$$I_{reg} \leftarrow Start$$

In the Mathcad routine, the first three equations inside the *n* loop are

$$I_{reg} \leftarrow A_t \cdot ELN_{ABC} - B_t \cdot I_{reg}$$

$$V2LN_{abc} \leftarrow A_{reg} \cdot VReg_{abc} - B_{reg} \cdot I_{abc}$$

$$V3LN_{abc} \leftarrow A_{line} \cdot V2LN_{abc} - B_{line} \cdot I_{abc}$$

At the end of the loop, the following equations are added:

$$I_{reg} \leftarrow d_{reg} \cdot I_{abc}$$

$$I_{ABC} \leftarrow d_t \cdot I_{reg}$$

With the three regulators installed, the load voltages on a 120 V base are

$$[Vload_{abc}] = \begin{bmatrix} 119.8 \\ 119.7 \\ 119.7 \end{bmatrix}$$

As can been seen from this example, as more elements of a system are added, there will be one equation for each of the system elements for the forward sweep and backward sweeps. This concept will be further developed in later chapters.

Delta-Grounded Wye Step-Up Connection

Fig.7 shows the connection diagram for the delta–grounded wye step-up connection.
Phasor diagrams for the voltages and currents are also shown in Fig.6. Note that the high-side line-to-line voltage leads the low-side line-to-line voltage and the same can be said for the high- and low-side line currents.

The development of the generalized matrices follows the same procedure as was used for the step-down connection. Only two matrices differ between the two connections.

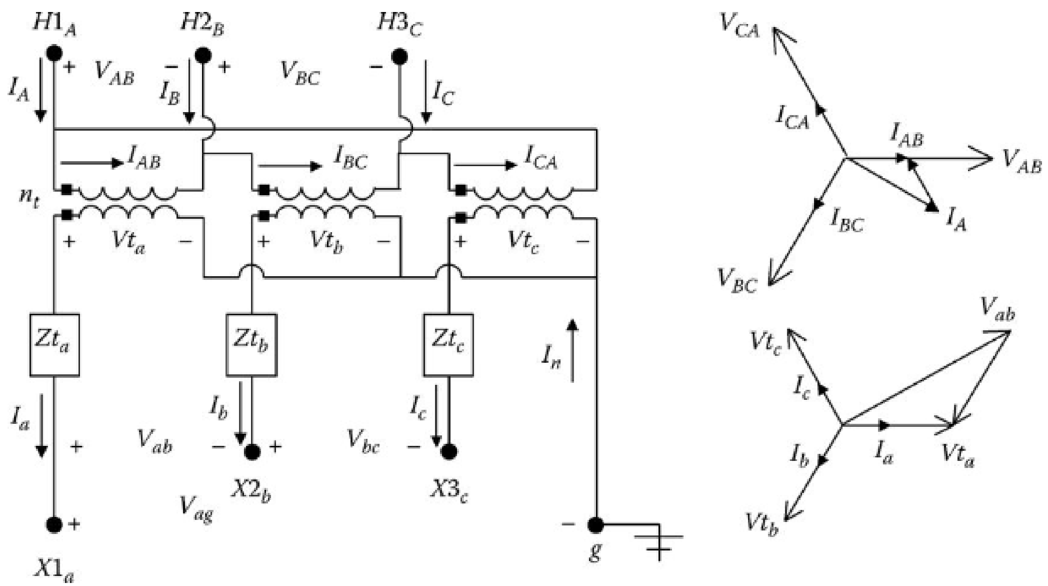
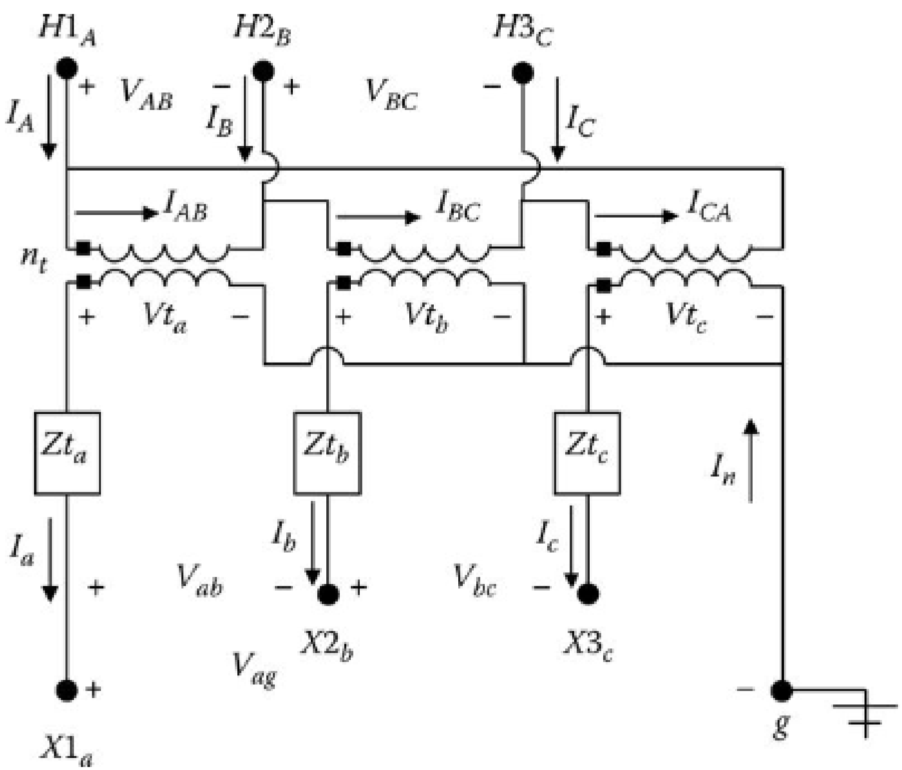
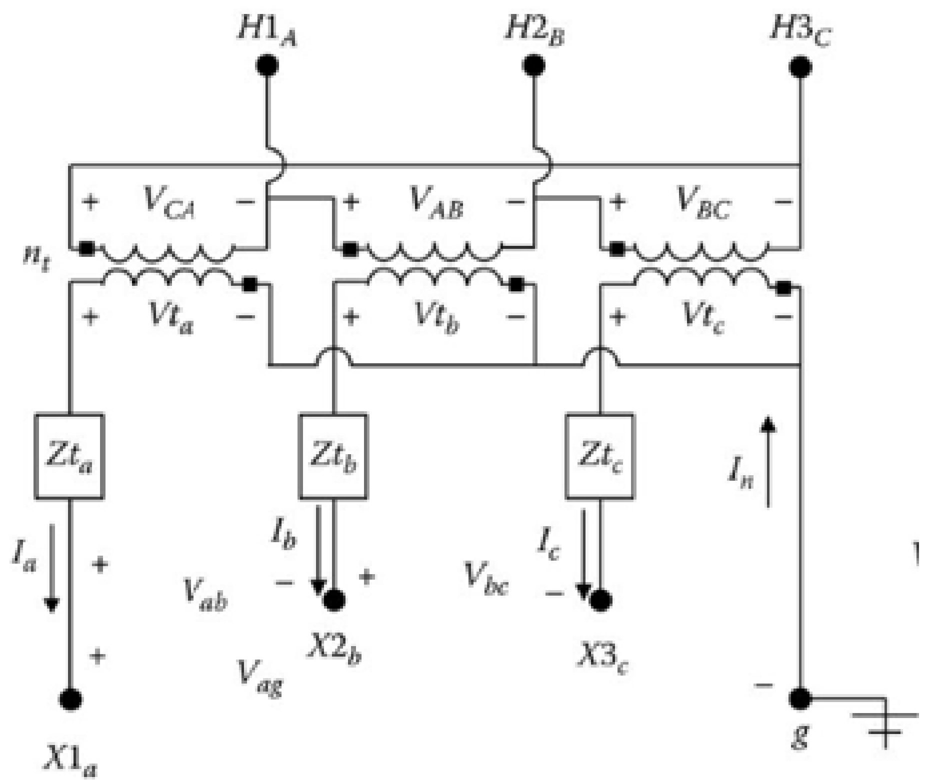


Fig.7 Delta-grounded wye step-up connection

Delta-Grounded Wye Step-Up VS Step-Down Connections



Delta-grounded wye step-up connection



Delta-grounded wye step-down connection

Delta-Grounded Wye Step-Up Connection

The primary (low side) line-to-line voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \quad [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{39}$$

where

$$[AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad n_t = \frac{KVLL_{rated\ Primary}}{KVLN_{rated\ Secondary}}$$

The primary delta currents are given by

$$\begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad [ID_{ABC}] = [AI] \cdot [I_{abc}] \tag{40}$$

where

$$[AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Delta-Grounded Wye Step-Up Connection

The primary line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \quad [I_{ABC}] = [DI] \cdot [ID_{ABC}] \tag{41}$$

where

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

The forward sweep matrices are

Applying Equation (28),

$$[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{42}$$

Applying Equation (31),

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{43}$$

Delta-Grounded Wye Step-Up Connection

The backward sweep matrices are

Applying Equation (19),

$$[a_t] = [W] \cdot [AV] = \frac{n_t}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{44}$$

Applying Equation (23),

$$[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{n_t}{3} \cdot \begin{bmatrix} 2 \cdot Zt_a & Zt_b & 0 \\ 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \end{bmatrix} \tag{45}$$

Applying Equation (37),

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{46}$$

Undergrounded Wye-Delta Step-Down Connection

Three single-phase transformers can be connected in a wye–delta connection. The neutral of the wye can be grounded or ungrounded. The grounded wye connection is characterized by

- The grounded wye provides a path for zero sequence currents for line-to-ground faults upstream from the transformer bank. This causes the transformers to be susceptible to burnouts on the upstream faults.
- If one phase of the primary circuit is opened, the transformer bank will continue to provide three-phase service by operating as an open wye–open delta bank. However, the two remaining transformers may be subjected to an overload condition leading to burnout.

The most common connection is the ungrounded wye–delta. This connection is typically used to provide service to a combination of single-phase "lighting" load and a three-phase "power" load such as an induction motor. The generalized constants for the ungrounded wye–delta transformer connection will be developed following the same procedure as was used for the delta–grounded wye.

Undergrounded Wye-Delta Step-Down Connection

Three single-phase transformers can be connected in an ungrounded-wye “standard 30 degree connection” as shown in Fig.8.

The voltage phasor diagrams in Fig.8 illustrate that the high-side positive sequence line-to-line voltage leads the low-side positive sequence line-to-line voltage by 30°. Also, the same phase shift occurs between the high-side line-to-neutral voltage and the low-side “equivalent” line-to-neutral voltage. The negative sequence phase shift is such that the high-side negative sequence voltage will lag the low-side negative sequence voltage by 30°.

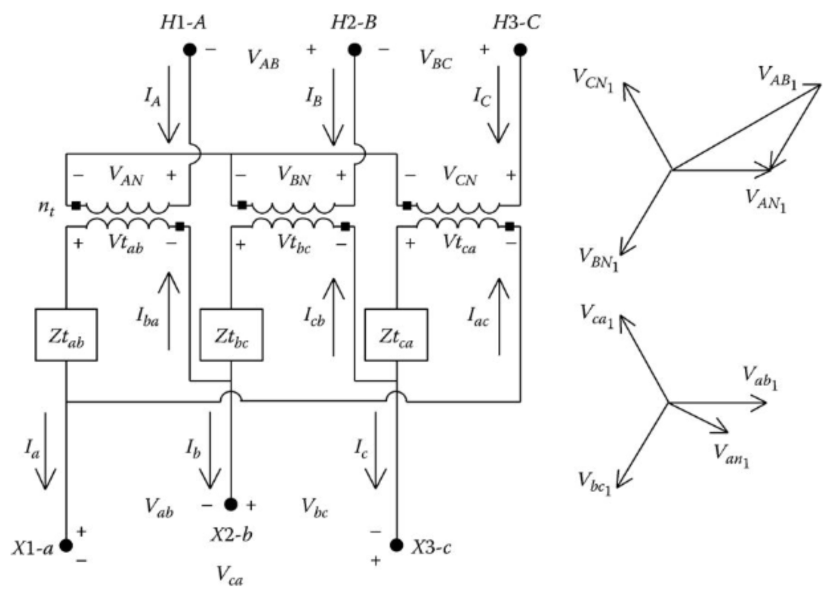


Fig.8 Standard ungrounded wye-delta connection step-down

Undergrounded Wye-Delta Step-Down Connection

The positive sequence current phasor diagrams for the connection in Fig.8 are shown in Fig.9.

Fig.9 illustrates that the positive sequence line current on the high side of the transformer (node ***n***) leads the low-side line current (node ***m***) by 30°. It can also be shown that the negative sequence high-side line current will lag the negative sequence low-side line current by 30°.

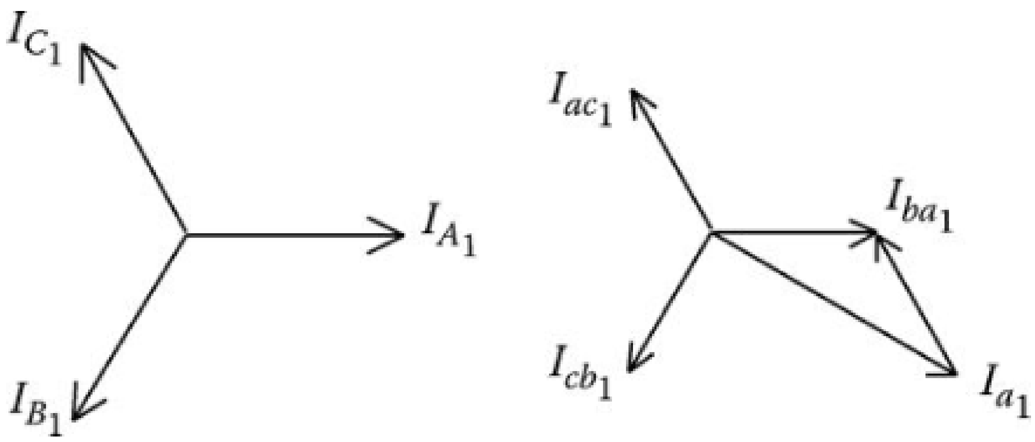


Fig.9 Positive sequence current phasors

Undergrounded Wye-Delta Step-Down Connection

The positive sequence current phasor diagrams for the connection in Fig.8 are shown in Fig.9.

Fig.9 illustrates that the positive sequence line current on the high side of the transformer (node ***n***) leads the low-side line current (node ***m***) by 30°. It can also be shown that the negative sequence high-side line current will lag the negative sequence low-side line current by 30°.

The definition for the "turns ratio n_t" will be the same as Equation (9) with the exception that the numerator will be the line-to-neutral voltage and the denominator will be the line-to-line voltage. It should be noted in Fig.8 that the "ideal" low-side transformer voltages for this connection will be line-to-line voltages. Also, the "ideal" low-side currents are the currents flowing inside the delta.

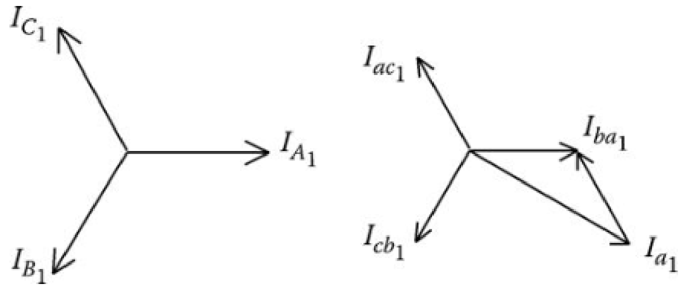


Fig.9 Positive sequence current phasors

Undergrounded Wye-Delta Step-Down Connection

The basic "ideal" transformer voltage and current equations as a function of the "turns ratio" are

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{47}$$

where

$$n_t = \frac{KVLN_{rated\ Primary}}{KVLL_{rated\ Secondary}}$$

$$[VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{48}$$

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{49}$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{50}$$

Undergrounded Wye-Delta Step-Down Connection

$$[VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{48}$$

Solving Equation (48) for the "ideal" delta transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \tag{51}$$

The line-to-line voltages at node ***m*** as a function of the "ideal" transformer voltages and the delta currents are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \tag{52}$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{53}$$

Undergrounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \quad (50)$$

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \quad (51)$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}] \quad (53)$$

Substitute Equations (50) and (51) into Equation (53):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \quad (54)$$

where

$$[ZNt_{abc}] = [Zt_{abc}] \cdot [AI] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & n_t \cdot Zt_{bc} & 0 \\ 0 & 0 & n_t \cdot Zt_{ca} \end{bmatrix} \quad (55)$$

The line currents on the delta side of the transformer bank as a function of the wye transformer currents are given by

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \quad (56)$$

where

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (57)$$

Undergrounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \quad (50)$$

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \quad (56)$$

Substitute Equation (50) into Equation (56):

$$[I_{abc}] = [DI] \cdot [AI] \cdot [I_{ABC}] = [DY] \cdot [I_{ABC}] \quad (58)$$

where

$$[DY] = [DI] \cdot [AI] = \begin{bmatrix} n_t & 0 & -n_t \\ -n_t & n_t & 0 \\ 0 & -n_t & n_t \end{bmatrix} \quad (59)$$

Undergrounded Wye-Delta Step-Down Connection

Because the matrix $[DY]$ is singular, it is not possible to use Equation (58) to develop an equation relating the wye-side line currents at node ***n*** to the delta-side line currents at node ***m***. In order to develop the necessary matrix equation, three independent equations must be written. Two independent KCL equations at the vertices of the delta can be used. Because there is no path for the high-side currents to flow to ground, they must sum to zero and, therefore, so must the delta currents in the transformer secondary sum to zero. This provides the third independent equation. The resulting three independent equations in matrix form are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{60}$$

Solving Equation (60) for the delta currents,

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 1 \\ -2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{61}$$

$$[ID_{abc}] = [L0] \cdot [I_{ab0}] \tag{62}$$

Undergrounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [L0] \cdot [I_{ab0}] \quad (62)$$

Equation (62) can be modified to include the phase c current by setting the third column of the $[L0]$ matrix to zero:

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (63)$$

$$[ID_{abc}] = [L] \cdot [I_{abc}] \quad (64)$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \quad (50)$$

Solve Equation (50) for $[I_{ABC}]$ and substitute into Equation (64):

$$[I_{ABC}] = [AI]^{-1} \cdot [L] \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \quad (65)$$

where

$$[d_t] = [AI]^{-1} \cdot [L] = \frac{1}{3 \cdot n_t}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \quad (66)$$

Undergrounded Wye-Delta Step-Down Connection

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{63}$$

$$[d_t] = [AI]^{-1} \cdot [L] = \frac{1}{3 \cdot n_t}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \tag{66}$$

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \tag{54}$$

Equation (66) defines the generalized constant matrix $[d_t]$ for the ungrounded wye–delta transformer connection. In the process of the derivation, a very convenient equation (Equation (63)) evolved that can be used anytime the currents in a delta need to be determined knowing the line currents. However, it must be understood that this equation will only work when the delta currents sum to zero, which means an ungrounded neutral on the primary.

The generalized matrices $[a_t]$ and $[b_t]$ can now be developed. Solve Equation (54) for $[VLN_{ABC}]$:

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [I_{ABC}] \tag{67}$$

Undergrounded Wye-Delta Step-Down Connection

$$[I_{ABC}] = [AI]^{-1} \cdot [L] \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \quad (65)$$

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [I_{ABC}] \quad (67)$$

Substitute Equation (65) into Equation (67):

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLL_{abc}] = [D] \cdot [VLN_{abc}]$$

where

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[VLN_{ABC}] = [AV] \cdot [D] \cdot [VLN_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{abc}]$$

where

$$[VLN_{abc}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \quad (68)$$

$$[a_t] = [AV] \cdot [D] = n_t \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (69)$$

$$[b_t] = [AV] \cdot [ZNt_{abc}] \cdot [d_t] = \frac{n_t}{3} \cdot \begin{bmatrix} Zt_{ab} & -Zt_{ab} & 0 \\ Zt_{bc} & 2 \cdot Zt_{bc} & 0 \\ -2 \cdot Zt_{ca} & -Zt_{ca} & 0 \end{bmatrix} \quad (70)$$

Undergrounded Wye-Delta Step-Down Connection

The generalized constant matrices have been developed for computing voltages and currents from the load toward the source (backward sweep). The forward sweep matrices can be developed by referring back to Equation (54), which is repeated here for convenience:

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \quad (71)$$

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \quad (16)$$

Equation (16) is used to compute the equivalent line-to-neutral voltages as a function of the line-to-line voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \quad (72)$$

Undergrounded Wye-Delta Step-Down Connection

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \quad (71)$$

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \quad (72)$$

Substitute Equation (71) into Equation (72):

$$[VLN_{abc}] = [W] \cdot [AV]^{-1} \cdot [VLN_{ABC}] - [W] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{ABC}] \quad (73)$$

where

$$[A_t] = [W] \cdot [AI]^{-1} = \frac{1}{3 \cdot n_t} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad (74)$$

$$[B_t] = [W] \cdot [ZNt_{abc}] \cdot [d_t] = \frac{1}{9} \begin{bmatrix} 2 \cdot Zt_{ab} + Zt_{bc} & 2 \cdot Zt_{bc} - Zt_{ab} & 0 \\ 2 \cdot Zt_{bc} - 2 \cdot Zt_{ca} & 4 \cdot Zt_{bc} - Zt_{ca} & 0 \\ Zt_{ab} - 4 \cdot Zt_{ca} & -Zt_{ab} - 2 \cdot Zt_{ca} & 0 \end{bmatrix} \quad (75)$$

The generalized matrices have been developed for the ungrounded wye–delta transformer connection. The derivation has applied basic circuit theory and the basic theories of transformers. The end result of the derivations is to provide an easy method of analyzing the operating characteristics of the transformer connection. Example 8.3 will demonstrate the application of the generalized matrices for this transformer connection.

Example 3

Fig.10 shows three single-phase transformers in an ungrounded wye–delta connection serving a combination of single-phase and three-phase load in a delta connection. The voltages at the load are balanced three phase of 240 V line to line. The net loading by phase is

$$S_{ab} = 100\ kVA\ at\ 0.9\ lagging\ power\ factor$$

$$S_{bc} = S_{ca} = 50\ kVA\ at\ 0.8\ lagging\ power\ factor$$

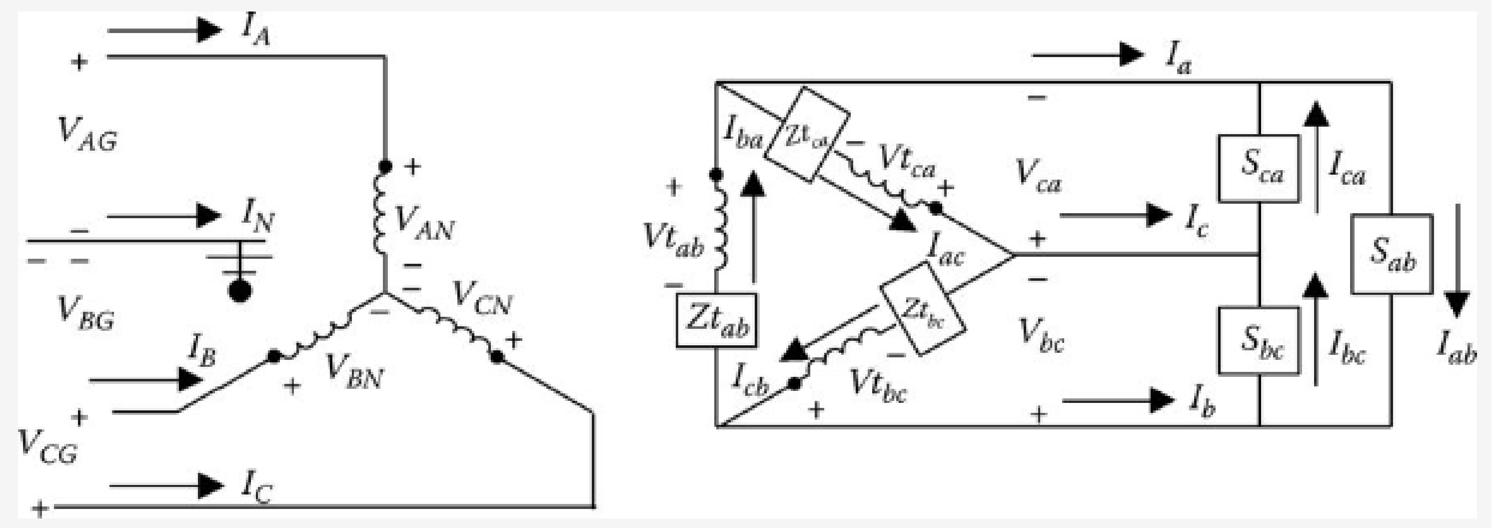


Fig.10 Undergrounded wye-delta step-down with unbalanced load

Example 3

The transformers are rated as follows:

- Phase *AN*: 100 kVA, 7200–240 V, $Z = 0.01 + j0.04$ per unit
- Phases *BN* and *CN*: 50 kVA, 7200–240 V, $Z = 0.015 + j0.035$ per unit

Determine the following:

- The currents in the load
- The secondary line currents
- The equivalent line-to-neutral secondary voltages
- The primary line-to-neutral and line-to-line voltages
- The primary line currents

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta-connected secondary windings.

"Lighting" transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576$$

$$Zt_{ab} = (0.01 + j0.4) \cdot 0.576 = 0.0058 + j0.023\ \Omega$$

"Power" transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152$$

$$Zt_{bc} = Zt_{ca} = (0.015 + j0.35) \cdot 1.152$$
$$= 0.0173 + j0.0403\ \Omega$$

Example 3

The transformer impedance matrix can now be defined:

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j0.023 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is n_t = 7200/240 = 30.

Define all of the matrices

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[a_t] = n_t \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 30 & -30 & 0 \\ 0 & 30 & -30 \\ -30 & 0 & 30 \end{bmatrix}$$

$$[b_t] = \frac{n_t}{3} \cdot \begin{bmatrix} Zt_{ab} & -Zt_{ab} & 0 \\ Zt_{bc} & 2Zt_{bc} & 0 \\ -2Zt_{ca} & -Zt_{ca} & 0 \end{bmatrix} = \begin{bmatrix} 0.0576 + j0.2304 & -0.576 - j0.2304 & 0 \\ 0.1728 + j0.4032 & 0.3456 + j0.8064 & 0 \\ -0.3456 - j0.8064 & -0.1728 - j0.4032 & 0 \end{bmatrix}$$

Example 3

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{3n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0111 & -0.0111 & 0 \\ 0 & 0.0111 & -0.0111 \\ -0.0111 & 0 & 0.0111 \end{bmatrix}$$

$$[A_t] = \frac{1}{3n_t} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.0222 & 0.0111 & 0 \\ 0 & 0.0222 & 0.0111 \\ 0.0111 & 0 & 0.0222 \end{bmatrix}$$

$$[B_t] = \frac{1}{9} \begin{bmatrix} 2Zt_{ab} + Zt_{bc} & 2Zt_{bc} - Zt_{ab} & 0 \\ 2Zt_{bc} - 2Zt_{ca} & 4Zt_{bc} - Zt_{ca} & 0 \\ Zt_{ab} - 4Zt_{ab} & -Zt_{ab} - 2Zt_{ab} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0032 + j0.0096 & 0.0026 + j0.0038 & 0 \\ 0 & 0.0058 + j0.0134 & 0 \\ -0.007 - j0.0154 & -0.0045 - j0.0115 & 0 \end{bmatrix}$$

Example 3

Define the line-to-line load voltages:

$$[VLL_{abc}] = \begin{bmatrix} 240\angle 0 \\ 240\angle -120 \\ 240\angle 120 \end{bmatrix}$$

Define the loads:

$$[SD_{abc}] = \begin{bmatrix} 100\angle \cos^{-1}(0.9) \\ 50\angle \cos^{-1}(0.8) \\ 50\angle \cos^{-1}(0.8) \end{bmatrix} = \begin{bmatrix} 90 + j43.589 \\ 30 + j30 \\ 30 + j30 \end{bmatrix} \text{kVA}$$

Calculate the delta load current

$$ID_i = \left(\frac{SD_i \cdot 1000}{VLL_{abc_i}}\right)^* A$$

$$[ID_{abc}] = \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \begin{bmatrix} 416.7\angle -25.84 \\ 208.3\angle -156.87 \\ 208.3\angle 83.13 \end{bmatrix} A$$

Example 3

Compute the secondary line current:

$$[I_{abc}] = [DI] \cdot [ID_{abc}] = \begin{bmatrix} 522.9\angle-47.97 \\ 575.3\angle-119.06 \\ 360.8\angle 53.13 \end{bmatrix} A$$

Compute the equivalent secondary line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] = \begin{bmatrix} 138.56\angle-30 \\ 138.56\angle-150 \\ 138.56\angle 90 \end{bmatrix} V$$

Use the generalized constant matrices to compute the primary line-to-neutral voltages and line-to-line voltages:

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] = \begin{bmatrix} 7367.6\angle 1.4 \\ 7532.3\angle-119.1 \\ 7406.2\angle 121.7 \end{bmatrix} V$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 12{,}9356\angle 31.54 \\ 12{,}8845\angle-88.95 \\ 12{,}8147\angle 151.50 \end{bmatrix} kV$$

Example 3

The high primary line currents are

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] = \begin{bmatrix} 11.54\angle -28.04 \\ 8.95\angle -166.43 \\ 7.68\angle 101.16 \end{bmatrix} A$$

It is interesting to compute the operating kVA of the three transformers. Taking the product of the transformer voltage times the conjugate of the current gives the operating kVA of each transformer.

$$ST_i = \frac{VLN_{ABC_i} \cdot \left(I_{ABC_i}\right)^*}{1000} = \begin{bmatrix} 85.02\angle 29.46 \\ 67.42\angle 47.37 \\ 56.80\angle 20.58 \end{bmatrix} kVA$$

The operating power factors of the three transformers are

$$[PF] = \begin{bmatrix} \cos(29.46) \\ \cos(47.39) \\ \cos(20.58) \end{bmatrix} = \begin{bmatrix} 87.1 \\ 67.7 \\ 93.6 \end{bmatrix} \%$$

Example 3

Note that the operating kVAs do not match very closely the rated kVAs of the three transformers. In particular, the transformer on phase *A* did not serve the total load of 100 kVA that is directly connected its terminals. That transformer is operating below rated kVA, while the other two transformers are overloaded. In fact, the transformer connected to phase *B* is operating 35% above rated kVA. Because of this overload, the ratings of the three transformers should be changed so that the phase *B* and phase *C* transformers are rated 75 kVA. Finally, the operating power factors of the three transformers bare little resemblance to the load power factors.

Example 3 demonstrates how the generalized constant matrices can be used to determine the operating characteristics of the transformers. In addition, the example shows that the obvious selection of transformer ratings will lead to an overload condition on the two power transformers. The beauty in this is that if the generalized constant matrices have been applied in a computer program, it is a simple task to change the transformer kVA ratings and be assured that none of the transformers will be operating in an overload condition.

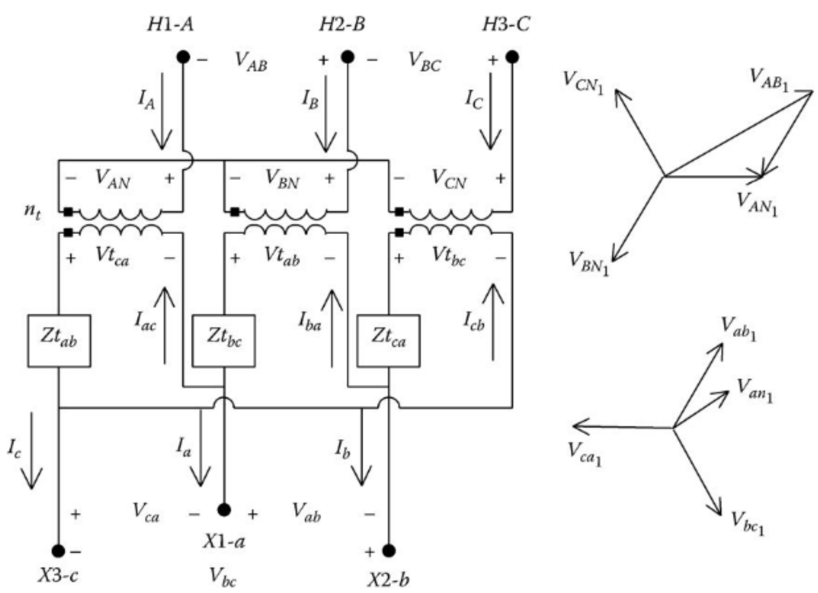
Example 3 has demonstrated the "backward" sweep to compute the primary voltages and currents. As before when the source (primary) voltages are given along with the load *PQ*, then the ladder iterative technique must be used to analyze the transformer connection.

Undergrounded Wye-Delta Step-up Connection

The connection diagram for the step-up connection is shown in Fig.11.

The only difference between the step-up and step-down connections are the definitions of the turns ratio n_t, $[AV]$, and $[AI]$. For the step-up connection,

$$n_t = \frac{KVLN_{rated\ Primary}}{KVLL_{rated\ Secondary}} \quad (76)$$



$$[AV] = n_t \cdot \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad (77)$$

$$[AI] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \quad (78)$$

Fig.11 Undergrounded wye-delta step-up connection

Example 5

The equations for the forward and backward sweep matrices, as defined in Section 8.3, can be applied using the definitions in Equations (76) through (78). The system of Example 8.3 is modified so that transformer connection is step-up. The transformers have the same ratings, but now the rated voltages for the primary and secondary are

$$n_t = \frac{KVLN_{rated\,Primary}}{KVLL_{rated\,Secondary}} \quad (76)$$

$$[AV] = n_t \cdot \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad (77)$$

$$[AI] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \quad (78)$$

Primary:

$$VLL_{pri} = 240 \qquad VLN_{pri} = 138.6\ V$$

Secondary:

$$VLL_{sec} = 12{,}470\ V$$

$$n_t = \frac{240}{12{,}470} = 0.0111$$

Example 5

The transformer impedances must be computed in Ohms relative to the secondary and then used to compute the new forward and backward sweep matrices. When this is done the new matrices are

$$[d_t] = [AI]^{-1} \cdot [L] = \begin{bmatrix} 60 & 30 & 0 \\ -30 & 30 & 0 \\ -30 & -60 & 0 \end{bmatrix}$$

$$[A_t] = [W] \cdot [AV]^{-1} = \begin{bmatrix} 0 & -60 & -30 \\ -30 & 0 & -60 \\ -60 & -30 & 0 \end{bmatrix}$$

$$[B_t] = [W] \cdot [ZNt_{abc}] \cdot [d_t] = \begin{bmatrix} 8.64 + j25.92 & 6.91 + j10.37 & 0 \\ 0 & 15.55 + j36.28 & 0 \\ -19.01 - j41.47 & -12.09 - j31.10 & 0 \end{bmatrix}$$

Using these matrices and the same loads, the output of the program gives the new load voltages as

$$[VLL_{abc}] = \begin{bmatrix} 12{,}055\angle 58.2 \\ 11{,}982\angle -61.3 \\ 12{,}106\angle 178.7 \end{bmatrix} V$$

Grounded Wye-Delta Step-Down Connection

The connection diagram for the standard 30 degree grounded wye (high)–delta (low) transformer connection grounded through an impedance of Z_g is shown in Fig.12. Note that the primary is grounded through an impedance Z_g.

Basic transformer equations:

The turn's ratio is given by

$$n_t = \frac{KVLN_{rated\ Primary}}{KVLL_{rated\ Secondary}} \qquad (79)$$

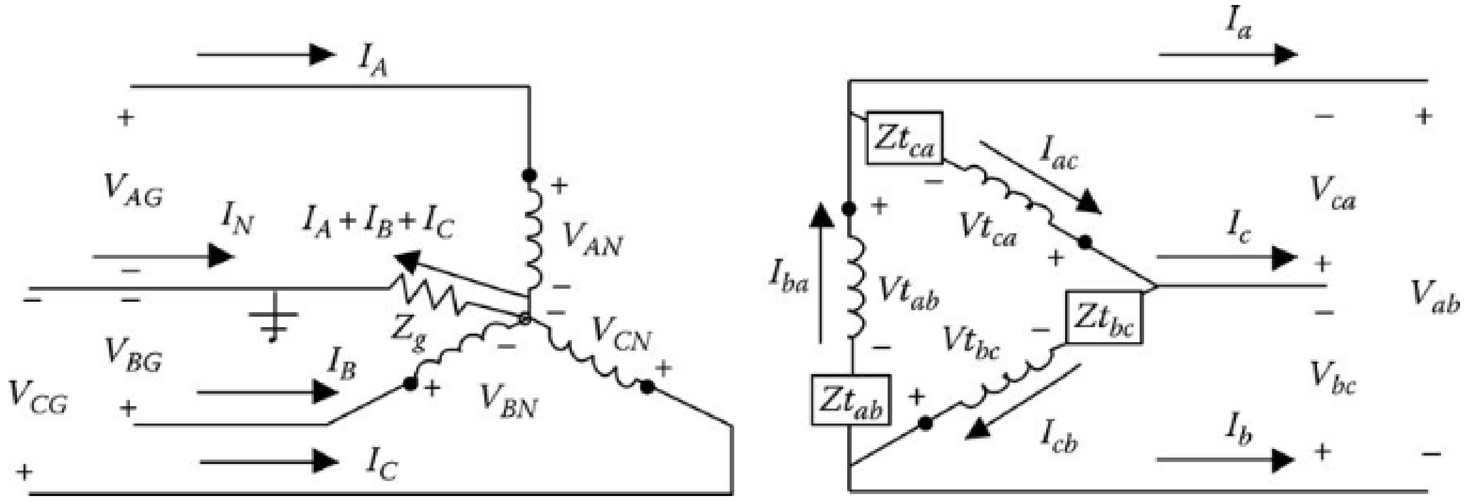


Fig.12 The grounded wye-delta connection

Grounded Wye-Delta Step-Down Connection

Basic transformer equations:

The basic "ideal" transformer voltage and current equations as a function of the turns ratio are

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \quad (80)$$

$$[VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad [AV] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (81)$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \qquad [AI] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Grounded Wye-Delta Step-Down Connection

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \quad (80)$$

Solving Equation (80) for the "ideal" transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \quad (82)$$

The line-to-neutral transformer primary voltages as a function of the system line-to-ground voltages are given by

$$V_{AN} = V_{AG} - Z_g \cdot (I_A + I_B + I_C)$$

$$V_{BN} = V_{BG} - Z_g \cdot (I_A + I_B + I_C)$$

$$V_{CN} = V_{CG} - Z_g \cdot (I_A + I_B + I_C)$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (83)$$

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}]$$

where $[ZG] = \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix}$

Grounded Wye-Delta Step-Down Connection

The line-to-line voltages on the delta side are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \quad (84)$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}]$$

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \quad (82)$$

Substitute Equation (82) into Equation (84):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [Zt_{abc}] \cdot [ID_{abc}] \quad (85)$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \quad (81)$$

Substitute Equation (81) into Equation (85):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - ([Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \quad (86)$$

Grounded Wye-Delta Step-Down Connection

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}] \tag{83}$$

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - ([Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \tag{86}$$

Substitute Equation (83) into Equation (86):

$$[VLL_{abc}] = [AV]^{-1} \cdot ([VLG_{ABC}] - [ZG] \cdot [I_{ABC}]) - ([Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}]$$

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - ([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \tag{87}$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{81}$$

Equation (81) gives the delta secondary currents as a function of the primary wye-side line currents. The secondary line currents are related to the secondary delta currents by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \tag{88}$$

$$[I_{abc}] = [DI] \cdot [ID_{abc}]$$

Grounded Wye-Delta Step-Down Connection

The real problem of transforming currents from one side to the other occurs for the case when the line currents on the delta secondary side $[I_{abc}]$ are known and the transformer secondary currents $[ID_{abc}]$ and primary line currents on the wye side $[I_{ABC}]$ are needed. The only way a relationship can be developed is to recognize that the sum of the line-to-line voltages on the delta secondary of the transformer bank must add up to zero. Three independent equations can be written as follows:

$$
\begin{aligned}
I_a &= I_{ba} - I_{ac} \\
I_b &= I_{cb} - I_{ba}
\end{aligned}
\tag{89}
$$

KVL around the delta secondary windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{90}$$

Replacing the “ideal” secondary delta voltages with the primary line-to-neutral voltages,

$$\frac{V_{AN}}{n_t} + \frac{V_{BN}}{n_t} + \frac{V_{CN}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{91}$$

Grounded Wye-Delta Step-Down Connection

$$\frac{V_{AN}}{n_t} + \frac{V_{BN}}{n_t} + \frac{V_{CN}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{91}$$

Multiply both sides of the Equation (91) by the turns ratio n_t:

$$V_{AN} + V_{BN} + V_{CN} = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \tag{92}$$

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}] \tag{83}$$

Determine the left side of Equation (92) as a function of the line-to-ground voltages using Equation (83):

$$\begin{aligned} V_{AN} + V_{BN} + V_{CN} &= V_{AG} + V_{BG} + V_{CG} - 3 \cdot Z_g \cdot (I_A + I_B + I_C) \\ V_{AN} + V_{BN} + V_{CN} &= V_{AG} + V_{BG} + V_{CG} - 3 \cdot \frac{1}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) \end{aligned} \tag{93}$$

Grounded Wye-Delta Step-Down Connection

$$V_{AN} + V_{BN} + V_{CN} = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \qquad (92)$$

$$V_{AN} + V_{BN} + V_{CN} = V_{AG} + V_{BG} + V_{CG} - 3 \cdot \frac{1}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) \qquad (93)$$

Substitute Equation (93) into Equation (92):

$$V_{AG} + V_{BG} + V_{CG} - \frac{3}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac}$$

$$V_{sum} = (n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ba} + (n_t \cdot Zt_{bc} + \frac{3}{n_t} \cdot Z_g) \cdot I_{cb} + (n_t \cdot Zt_{ca} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ac}$$

$$V_{sum} = V_{AG} + V_{BG} + V_{CG} \qquad (94)$$

Grounded Wye-Delta Step-Down Connection

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \quad (88)$$

$$I_a = I_{ba} - I_{ac}$$
$$I_b = I_{cb} - I_{ba} \quad (89)$$

$$V_{sum} = (n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ba} + (n_t \cdot Zt_{bc} + \frac{3}{n_t} \cdot Z_g) \cdot I_{cb} + (n_t \cdot Zt_{ca} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ac} \quad (94)$$

Equations (88), (89), and (94) can be put into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ V_{sum} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (95)$$

Equations (95) in general form is

$$[X] = [F] \cdot [ID_{abc}] \quad (96)$$

Solve for $[ID_{abc}]$

$$[ID_{abc}] = [F]^{-1} \cdot [X] = [G] \cdot [X] \quad (97)$$

Grounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [F]^{-1} \cdot [X] = [G] \cdot [X] \tag{97}$$

Equations (97) in full form is

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ V_{AG} + V_{AG} + V_{AG} \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{98}$$

Equations (98) in short form is

$$[ID_{abc}] = [G1] \cdot [VLG_{ABC}] + [G2] \cdot [I_{abc}] \tag{99}$$

Grounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{81}$$

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{98}$$

Substitute Equation (81) into Equation (98):

$$[I_{ABC}] = [AI]^{-1} \cdot [ID_{abc}] = [AI]^{-1} \cdot ([G1] \cdot [VLG_{ABC}] + [G2] \cdot [I_{abc}]) \tag{100}$$

$$[I_{ABC}] = [x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}]$$

where

$$[x_t] = [AI]^{-1} \cdot [G1]$$

$$[d_t] = [AI]^{-1} \cdot [G2]$$

Equation (100) is used in the "backward" sweep to compute the primary currents based upon the secondary currents and primary *LG* voltages.

Grounded Wye-Delta Step-Down Connection

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - ([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \quad (87)$$

$$[I_{ABC}] = [x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}] \quad (100)$$

The "forward" sweep equation is determined by substituting Equation (100) into Equation (87):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - ([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]) \cdot ([x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}])$$

Define

$$[X1] = [Zt_{abc}] \cdot [AI] + [AV]^{-1} \cdot [ZG]$$

$$[VLL_{abc}] = ([AI] - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [X1] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}]$$

$$[VLN_{abc}] = [W] \cdot (([AV]^{-1} - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [X1] \cdot [d_t] \cdot [I_{abc}])$$

$$[VLN_{abc}] = [W] \cdot ([AV]^{-1} - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [W] \cdot [X1] \cdot [d_t] \cdot [I_{abc}] \quad (101)$$

Grounded Wye-Delta Step-Down Connection

$$[VLN_{abc}] = [W] \cdot ([AV]^{-1} - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [W] \cdot [X1] \cdot [d_t] \cdot [I_{abc}] \tag{101}$$

The final form of Equation (101) gives the equation for the forward sweep:

$$[VLN_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}] \tag{102}$$

where

$$[A_t] = [W] \cdot ([AV]^{-1} - [X1] \cdot [x_t])$$

$$[B_t] = [W] \cdot [X1] \cdot [d_t]$$

Open Wye-Open Delta

A common load to be served on a distribution feeder is a combination of a single-phase lighting load and a three-phase power load. Many times the three-phase power load will be an induction motor. This combination load can be served by a grounded or ungrounded wye–delta connection as previously described or by an "open wye–open delta" connection. When the three-phase load is small compared to the single-phase load, the open wye–open delta connection is commonly used. The open wye–open delta connection requires only two transformers, but the connection will provide three-phase line-to-line voltages to the combination load. Fig.13 shows the open wye–open delta connection and the primary and secondary positive sequence voltage phasors.

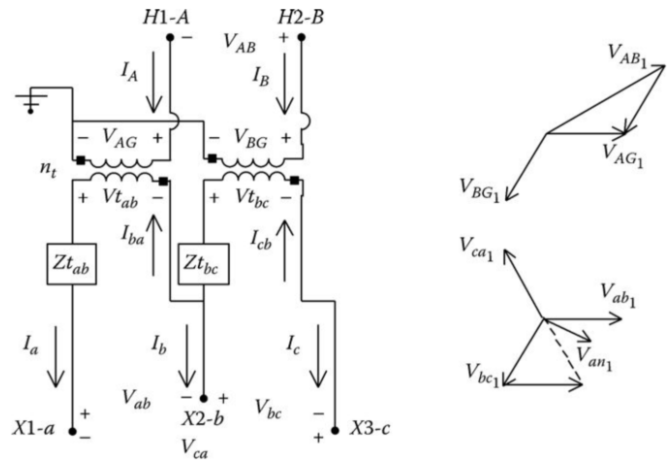


Fig.13 Open wye-open delta connection

Open Wye-Open Delta

With reference to Fig.13, the basic “ideal” transformer voltages as a function of the “turn's ratio” are

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \qquad (103)$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}]$$

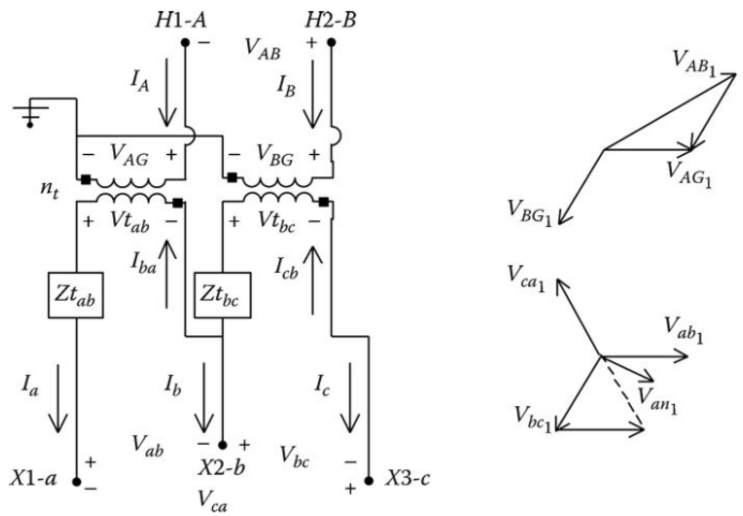


Fig.13 Open wye-open delta connection

Open Wye-Open Delta

The currents as a function of the turn's ratio are given by

$$I_{ba} = n_T \cdot I_A = I_a$$
$$I_{cb} = n_T \cdot I_B = -I_c \qquad (104)$$
$$I_b = -(I_a + I_c)$$

Equation (104) can be expressed in matrix form by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ -n_t & n_t & 0 \\ 0 & -n_t & 0 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (105)$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}]$$

The ideal voltages in the secondary can be determined by

$$Vt_{ab} = V_{ab} + Zt_{ab} \cdot I_a$$
$$Vt_{bc} = V_{bc} - Zt_{bc} \cdot I_c \qquad (106)$$

Open Wye-Open Delta

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{103}$$

$$\begin{aligned} Vt_{ab} &= V_{ab} + Zt_{ab} \cdot I_a \\ Vt_{bc} &= V_{bc} - Zt_{bc} \cdot I_c \end{aligned} \tag{106}$$

Substitute Equation (106) into Equation (103):

$$\begin{aligned} V_{AG} &= n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \\ V_{BG} &= n_t \cdot Vt_{bc} = n_T \cdot V_{bc} - n_t \cdot Zt_{bc} \cdot I_c \end{aligned} \tag{107}$$

Equation (107) can be put into three-phase matrix form as

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{108}$$

$$[VLG_{ABC}] = [AV] \cdot [VLL_{abc}] + [b_t] \cdot [I_{abc}]$$

Open Wye-Open Delta

$$[VLG_{ABC}] = [AV] \cdot [VLL_{abc}] + [b_t] \cdot [I_{abc}] \tag{108}$$

The secondary line-to-line voltages in Equation (108) can be replaced by the equivalent line-to-neutral secondary voltages:

$$[VLG_{ABC}] = [AV] \cdot [D] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$

$$[VLG_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{109}$$

where

$$[a_t] = [AV] \cdot [D] \qquad [b_t] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix}$$

The source-side line currents as a function of the load-side line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad [d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \tag{110}$$

Open Wye-Open Delta

$$[VLG_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{109}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \tag{110}$$

$$\begin{aligned} V_{AG} &= n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \\ V_{BG} &= n_t \cdot Vt_{bc} = n_T \cdot V_{bc} - n_t \cdot Zt_{bc} \cdot I_c \end{aligned} \tag{107}$$

Equations (109) and (110) give the matrix equations for the backward sweep. The forward sweep equation can be determined by solving Equation (107) for the two line-to-line secondary voltages:

$$\begin{aligned} V_{ab} &= \frac{1}{n_t} \cdot V_{AG} - Zt_{ab} \cdot I_a \\ V_{bc} &= \frac{1}{n_t} \cdot V_{BG} - Zt_{bc} \cdot I_c \end{aligned} \tag{111}$$

The third line-to-line voltage V_{ca} must be equal to the negative sum of the other two line-to-line voltages (KVL). In matrix form, the desired equation is

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & \frac{1}{n_t} & 0 \\ -\frac{1}{n_t} & -\frac{1}{n_t} & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & Zt_{bc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{112}$$

$$[VLL_{ab}] = [BV] \cdot [VLG_{ABC}] - [Zt_{abc}] \cdot [I_{abc}]$$

Open Wye-Open Delta

The equivalent secondary line-to-neutral voltages are then given by

$$[VLN_{abc}] = [W][VLL_{ABC}] = [W] \cdot [BV] \cdot [VLG_{ABC}] - [W] \cdot [Zt_{abc}] \ \cdot [I_{abc}] \tag{113}$$

The forward sweep equation is given by

$$[VLN_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}]$$

where

$$[A_t] = [W] \cdot [BV] = \frac{1}{3 \cdot n_t} \cdot \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

$$[B_t] = [W] \cdot [Zt_{abc}] = \frac{1}{3} \cdot \begin{bmatrix} 2 \cdot Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & 2 \cdot Zt_{bc} \end{bmatrix} \tag{114}$$

Open Wye-Open Delta

The open wye–open delta connection derived in this section utilized phases *A* and *B* on the primary. This is just one of three possible connections. The other two possible connections would use phases *B* and *C* and then phases *C* and *A*. The generalized matrices will be different from those just derived. The same procedure can be used to derive the matrices for the other two connections.

The terms "leading" and "lagging" connection are also associated with the open wye–open delta connection. When the lighting transformer is connected across the leading of the two phases, the connection is referred to as the "leading" connection. Similarly, when the lighting transformer is connected across the lagging of the two phases, the connection is referred to as the "lagging" connection. For example, if the bank is connected to phases *A* and *B* and the lighting transformer is connected from phase *A* to ground, this would be referred to as the "leading" connection because the voltage *A-G* leads the voltage *B-G* by 120°. Reverse the connection and it would now be called the "lagging" connection. Obviously, there is a leading and lagging connection for each of the three possible open wye–open delta connections.

Grounded Wye- Grounded Wye Connection

The grounded wye–grounded wye connection is primarily used to supply single-phase and three-phase loads on four-wire multigrounded systems. The grounded wye–grounded wye connection is shown in Fig.14.

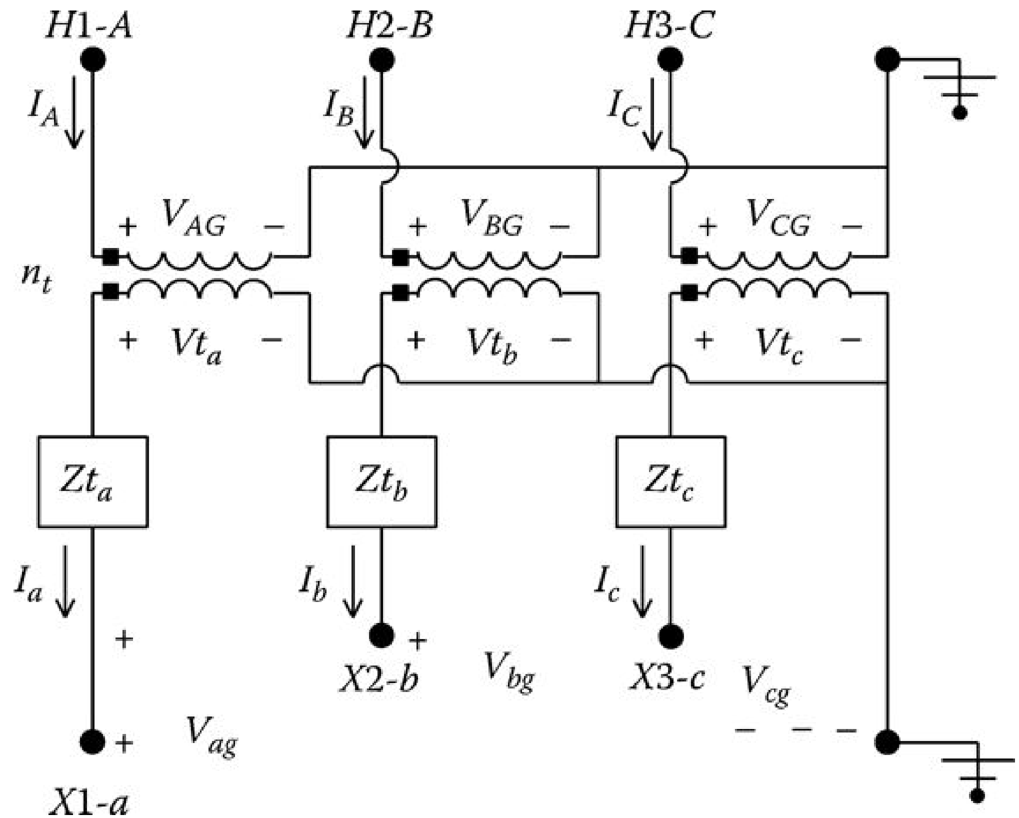


Fig.14 Grounded wye-grounded wye connection

Grounded Wye- Grounded Wye Connection

Unlike the delta–wye and wye–delta connections, there is no phase shift between the voltages and the currents on the two sides of the bank. This makes the derivation of the generalized constant matrices much easier. The ideal transformer equations are

$$n_t = \frac{KVLN_{rated\ Primary}}{KVLL_{rated\ Secondary}} \tag{115}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \tag{116}$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad [AV] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{117}$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \qquad [AI] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Grounded Wye- Grounded Wye Connection

With reference to Fig.14, the ideal transformer voltages on the secondary windings can be computed by

$$\begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} + \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (118)$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}]$$

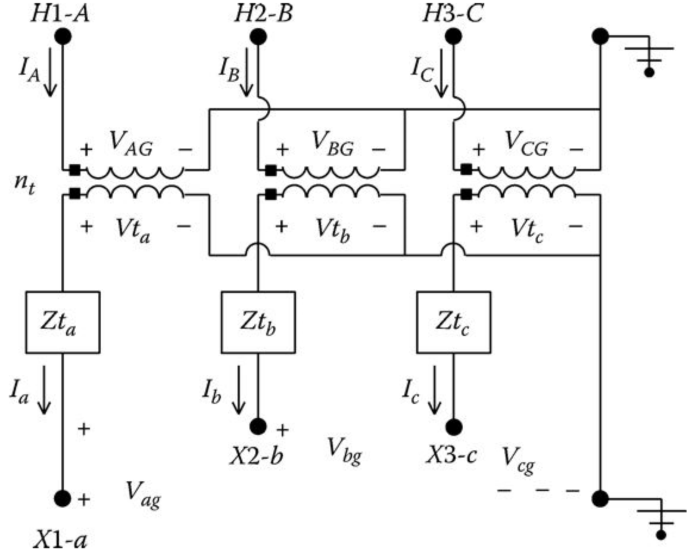


Fig.14 Grounded wye-grounded wye connection

Grounded Wye- Grounded Wye Connection

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{116}$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{118}$$

Substitute Equation (118) into Equation (116):

$$[VLG_{ABC}] = [AV] \cdot ([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}])$$

$$[VLG_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{119}$$

Equation (119) is the backward sweep equation with the $[a_t]$ and $[b_t]$ matrices defined by

$$[a_t] = [AV] = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \tag{120}$$

$$[b_t] = [AV] \cdot [Zt_{abc}] = \begin{bmatrix} n_t \cdot Zt_a & 0 & 0 \\ 0 & n_t \cdot Zt_b & 0 \\ 0 & 0 & n_t \cdot Zt_c \end{bmatrix} \tag{121}$$

Grounded Wye- Grounded Wye Connection

$$[VLG_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \quad (119)$$

The primary line currents as a function of the secondary line currents are given by

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \quad (122)$$

where

$$[d_t] = [AI]^{-1} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The forward sweep equation is determined solving Equation (119) for the secondary line-to-ground voltages:

$$[VLG_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - [Zt_{abc}] \cdot [I_{abc}] \quad (123)$$

$$[VLG_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}]$$

$$[A_t] = [AV]^{-1} \qquad [B_t] = [Zt_{abc}]$$

The modeling and analysis of the grounded wye–grounded wye connection does not present any problems. Without the phase shift there is a direct relationship between the primary and secondary voltages and currents as had been demonstrated in the derivation of the generalized constant matrices.

Delta-Delta Connection

The delta–delta connection is primarily used on three-wire delta systems to provide service to a three-phase load or a combination of three-phase and single-phase loads. Three single-phase transformers connected in a delta–delta are shown in Fig.15.

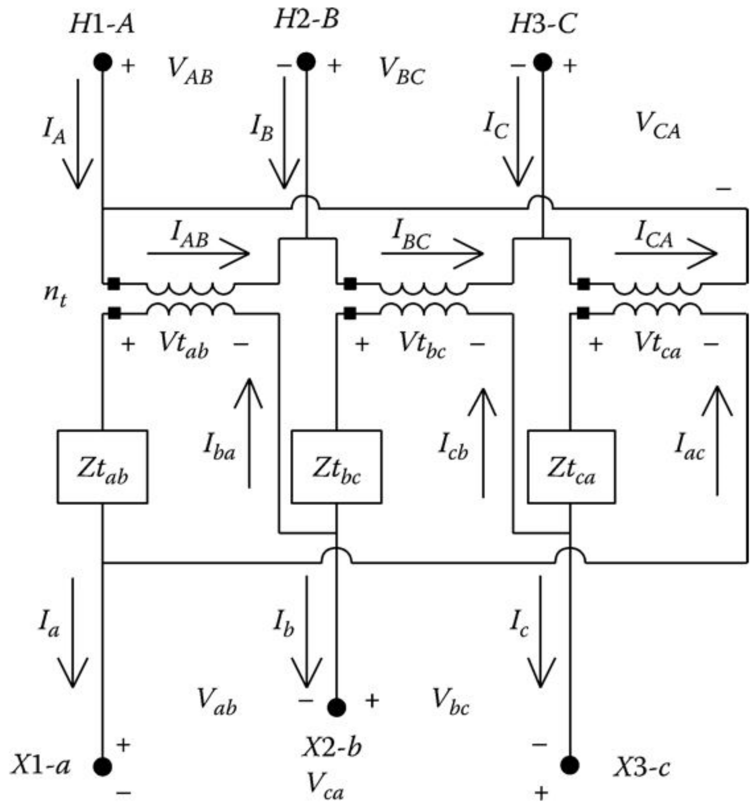


Fig.15 Grounded wye-grounded wye connection

Delta-Delta Connection

The basic ideal transformer voltage and current equations as a function of the "turn's ration" are

$$n_t = \frac{VLL_{rated\ Primary}}{VLL_{rated\ Secondary}} \tag{124}$$

$$\begin{bmatrix} VLL_{AB} \\ VLL_{BC} \\ VLL_{CA} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{125}$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ca} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \tag{126}$$

$$[ID_{abc}] = [AI] \cdot [ID_{ABC}] \qquad [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Delta-Delta Connection

$$[ID_{abc}] = [AI] \cdot [ID_{ABC}] \tag{126}$$

Solve Equation (126) for the source-side delta currents:

$$[ID_{ABC}] = [AI]^{-1} \cdot [ID_{abc}] \tag{127}$$

The line currents as a function of the delta currents on the source side are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix}$$

$$[I_{ABC}] = [DI] \cdot [ID_{ABC}] \tag{128}$$

where

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Substitute Equation (127) into Equation (128)

$$[I_{ABC}] = [DI] \cdot [AI]^{-1} \cdot [ID_{abc}] \tag{129}$$

Delta-Delta Connection

$$[I_{ABC}] = [DI] \cdot [AI]^{-1} \cdot [ID_{abc}] \tag{129}$$

Since $[AI]$ is a diagonal matrix, Equation (129) can be rewritten as

$$[I_{ABC}] = [AI]^{-1} \cdot [DI] \cdot [ID_{abc}] \tag{130}$$

The load-side line currents as a function of the load-side delta currents:

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \tag{131}$$

Applying Equation (131), Equation (130) becomes

$$[I_{ABC}] = [AI]^{-1} \cdot [I_{abc}] \tag{132}$$

Turn Equation (132) around to solve for the load-side line currents as a function of the source-side line currents:

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{133}$$

Equations (132) and (133) merely demonstrate that the line currents on the two sides of the transformer are in phase and differ only by the turn's ratio of the transformer windings. In the per-unit system, the per-unit line currents on the two sides of the transformer are exactly equal.

Delta-Delta Connection

The ideal delta voltages on the secondary side as a function of the line-to-line voltages, the delta currents, and the transformer impedances are given by

$$[Vt_{abc}] = [VLL_{abc}] + [Zt_{abc}] \cdot [ID_{abc}] \tag{134}$$

where

$$[Zt_{abc}] = \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix}$$

Substitute Equation (134) into Equation (125)

$$[VLL_{abc}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [ID_{abc}] \tag{135}$$

Solve Equation (135) for the load-side line-to-line voltages:

$$[VLL_{abc}] = [AI]^{-1} \cdot [VLL_{ABC}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{136}$$

The delta currents $[ID_{abc}]$ in Equations (135) and (136) need to be replaced by the secondary line currents $[I_{abc}]$.

Delta-Delta Connection

In order to develop the needed relationship, three independent equations are needed. The first two come from applying KCL at two vertices of the delta connected secondary:

$$I_a = I_{ba} - I_{ac}$$
$$I_b = I_{cb} - I_{ba} \tag{137}$$

The third equation comes from recognizing that the sum of the primary line-to-line voltages and therefore the secondary ideal transformer voltages must sum to zero. KVL around the delta windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{138}$$

Replacing the "ideal" delta voltages with the source-side line-to-line voltages,

$$\frac{V_{AB}}{n_t} + \frac{V_{BC}}{n_t} + \frac{V_{CA}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{139}$$

Since the sum of the line-to-line voltages must equal zero (KVL) and the turn's ratios of the three transformers are equal, Equation (139) is simplified to

$$0 = Zt_a \cdot I_{ba} + Zt_b \cdot I_{cb} + Zt_c \cdot I_{ac} \tag{140}$$

Delta-Delta Connection

$$0 = Zt_a \cdot I_{ba} + Zt_b \cdot I_{cb} + Zt_c \cdot I_{ac} \tag{140}$$

Note in Equation (140) that if the three transformer impedances are equal, then the sum of the delta currents will add to zero, meaning that the zero sequence delta currents will be zero.

Equations (137) and (140) can be put into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Zt_{ab} & Zt_{bc} & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

$$[I0_{abc}] = [F] \cdot [ID_{abc}] \tag{141}$$

where

$$[I0_{abc}] = \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Zt_{ab} & Zt_{bc} & Zt_{ca} \end{bmatrix}$$

Delta-Delta Connection

$$[I0_{abc}] = [F] \cdot [ID_{abc}] \tag{141}$$

Solve Equation (141) for the load-side delta currents:

$$[ID_{abc}] = [F]^{-1} \cdot [I0_{abc}] = [G] \cdot [I0_{abc}] \tag{142}$$

where

$$[G] = [F]^{-1}$$

Writing Equation (142) in matrix form gives

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{143}$$

From Equations (142) and (143), it is seen that the delta currents are a function of the transformer impedances and just the line currents in phases *a* and *b*. Equation (143) can be modified to include the line current in phase *c* by setting the last column of the [*G*] matrix to zeros:

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad [ID_{abc}] = [G1] \cdot [I_{abc}] \quad [G1] = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \tag{144}$$

Delta-Delta Connection

$$[VLL_{abc}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [ID_{abc}] \quad (135)$$

$$[ID_{abc}] = [G1] \cdot [I_{abc}] \quad (144)$$

When the impedances of the transformers are equal, the sum of the delta currents will be zero meaning that there is no circulating zero sequence current in the delta windings.
Substitute Equation (144) into Equation (135):

$$[VLL_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \quad (145)$$

The generalized matrices are defined in terms of the line-to-neutral voltages on the two sides of the transformer bank. Equation (145) is modified to be in terms of equivalent line-to-neutral voltages:

$$\begin{aligned} [VLN_{ABC}] &= [W] \cdot [VLL_{ABC}] \\ &= [W] \cdot [AV] \cdot [D] \cdot [VLN_{abc}] + [W] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \end{aligned} \quad (146)$$

Equation (146) is in the general form

$$\begin{aligned} [VLN_{ABC}] &= [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \\ [a_t] &= [W] \cdot [AV] \cdot [D] \\ [b_t] &= [AV] \cdot [W] \cdot [Zt_{abc}] \cdot [G1] \end{aligned} \quad (147)$$

Delta-Delta Connection

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{133}$$

Equation (133) gives the generalized equation for currents:

$$[I_{ABC}] = [AI]^{-1} \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \tag{148}$$

$$[d_t] = [AI]^{-1}$$

The forward sweep equations can be derived by modifying Equation (136) in terms of equivalent line-to-neutral voltages:

$$\begin{aligned}[VLN_{abc}] &= [W] \cdot [VLL_{ABC}] \\ &= [W] \cdot [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] - [W] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}]\end{aligned} \tag{149}$$

The forward sweep equation is

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{150}$$

where

$$[A_t] = [W] \cdot [AV]^{-1} \cdot [D]$$

$$[B_t] = [W] \cdot [Zt_{abc}] \cdot [G1]$$

The forward and backward sweep matrices for the delta–delta connection have been derived. Once again it has been a long process to get to the final six equations that define the matrices. The derivation provides an excellent exercise in the application of basic transformer theory and circuit theory. Once the matrices have been defined for a particular transformer connection, the analysis of the connection is a relatively simple task. Example 8 will demonstrate the analysis of this connection using the generalized matrices.

Example 8

Fig.16 shows three single-phase transformers in a delta–delta connection serving an unbalanced three-phase load connected in delta.

The source voltages at the load are balanced three phase of 240 V line to line:

$$[VLL_{abc}] = \begin{bmatrix} 12{,}470\angle 0 \\ 12{,}470\angle -120 \\ 12{,}470\angle 120 \end{bmatrix} V$$

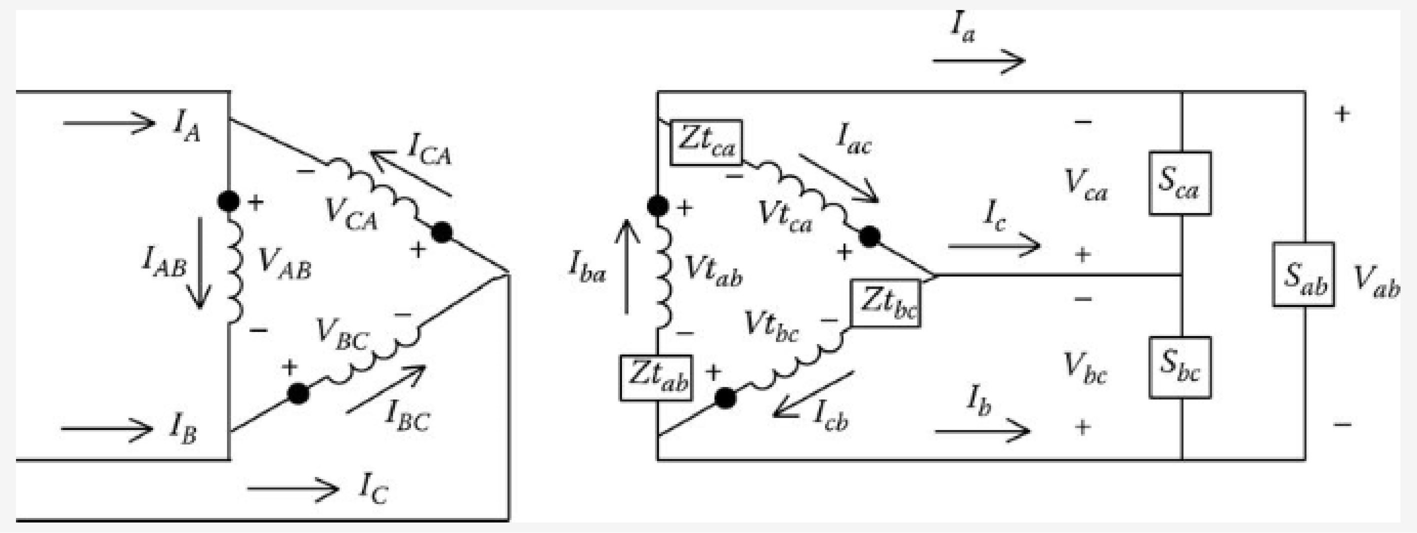


Fig.16 Delta-delta bank serving an unbalanced delta connected load

Example 8

The loading by phase is

S_{ab} = 100 kVA at 0.9 lagging power factor

$S_{bc} = S_{ca}$ = 50 kVA at 0.8 lagging power factor

The ratings of the transformers are

- Phase *AB*: 100 kVA, 12,470–240 V, $Z = 0.01 + j0.04$ per unit
- Phases *BC* and *CA* 50 kVA, 12,470–240 V, $Z = 0.015 + j0.035$ per unit

Determine the following:

1. The load line-to-line voltages
2. The secondary line currents
3. The primary line currents
4. The load currents
5. Load voltage unbalance

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta connected secondary windings.

Phase ab transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576\ \Omega$$

$$Zt_{ab} = (0.01 + j0.04) \cdot 0.576 = 0.0058 + j0.023\ \Omega$$

Example 8

Phase bc and ca transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152\ \Omega$$

$$Zt_{bc} = Zt_{ca} = (0.015 + j0.035) \cdot 1.152 = 0.0173 + j0.0403\ \Omega$$

The transformer impedance matrix can now be defined

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j0.23 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is $n_t = 12{,}470/240 = 51.9583$.
Define all of the matrices:

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[AV] = [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 51.9583 & 0 & 0 \\ 0 & 51.9583 & 0 \\ 0 & 0 & 51.9583 \end{bmatrix}$$

Example 8

$$[F] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0.0058 + j0.023 & 0.0173 + j0.0403 & 0.0173 + j0.0404 \end{bmatrix}$$

$$[G] = [F]^{-1} = \begin{bmatrix} 0.3941 - j0.0134 & -0.3941 + j0.0134 & 3.2581 - j8.378 \\ 0.3941 - j0.0134 & 0.6059 + j0.0134 & 3.2581 - j8.378 \\ -0.6059 - j0.0134 & -0.3941 + j0.0134 & 3.2581 - j8.378 \end{bmatrix}$$

$$[G1] = \begin{bmatrix} 0.3941 - j0.0134 & -0.3941 + j0.0134 & 0 \\ 0.3941 - j0.0134 & 0.6059 + j0.0134 & 0 \\ -0.6059 - j0.0134 & -0.3941 + j0.0134 & 0 \end{bmatrix}$$

$$[a_t] = [W] \cdot [AV] \cdot [D] = \begin{bmatrix} 34.6489 & -17.3194 & -17.3194 \\ -17.3194 & 34.6489 & -17.3194 \\ -17.3194 & -17.3194 & 34.6489 \end{bmatrix}$$

$$[b_t] = [AV] \cdot [W] \cdot [Zt_{abc}] \cdot [G1] = \begin{bmatrix} 0.2166 + j0.583 & 0.0826 + j0.1153 & 0 \\ 0.0826 + j0.1153 & 0.2166 + j0.583 & 0 \\ -0.2993 - j0.6983 & -0.2993 - j0.6983 & 0 \end{bmatrix}$$

Example 8

$$[d_t] = [AI]^{-1} = \begin{bmatrix} 0.0192 & 0 & 0 \\ 0 & 0.0192 & 0 \\ 0 & 0 & 0.0192 \end{bmatrix}$$

$$[A_t] = [W] \cdot [AV]^{-1} \cdot [D] = \begin{bmatrix} 0.0128 & -0.0064 & -0.0064 \\ -0.0064 & 0.0128 & -0.0064 \\ -0.0064 & -0.0064 & 0.0128 \end{bmatrix}$$

$$[B_t] = [W] \cdot [Zt_{abc}] \cdot [G1] = \begin{bmatrix} 0.0042 + j0.0112 & 0.0016 + j0.0022 & 0 \\ 0.0016 + j0.0022 & 0.0042 + j0.0112 & 0 \\ -0.0058 - j0.0134 & -0.0058 - j0.0134 & 0 \end{bmatrix}$$

Example 8

The Mathcad program is modified slightly to account for the delta connections. The modified program is shown in Fig.17.

$$\left|VLL_{ABC_i}\right| = \begin{pmatrix} 12470 \\ 12470 \\ 12470 \end{pmatrix} \qquad \frac{\arg\left(VLL_{ABC_i}\right)}{deg} = \begin{pmatrix} 30 \\ -90 \\ 150 \end{pmatrix}$$

$$Start := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad Tol := .000001 \qquad VM := kVLL_{sec} \cdot 1000 \qquad VM = 240$$

```
X := | I_abc ← Start
     | V_old ← Start
     | VLG_ABC ← W·VLL_ABC
     | for n ∈ 1..200
     |   | VLN_abc ← A_t·VLG_ABC − B_t·I_abc
     |   | VLL_abc ← D·VLN_abc
     |   | for j ∈ 1..3
     |   |   ID_abc_j ← (SL_j·1000 / VLL_abc_j)‾
     |   | for k ∈ 1..3
     |   |   Error_k ← |VLL_abc_k − V_old_k| / VM
     |   | Error_max ← max(Error)
     |   | break if Error_max < Tol
     |   | V_old ← VLL_abc
     |   | I_abc ← DI·ID_abc
     |   | I_ABC ← d_t·I_abc
     | Out_1 ← VLN_abc
     | Out_2 ← VLL_abc
     | Out_3 ← I_abc
     | Out_4 ← I_ABC
     | Out_5 ← ID_abc
     | Out_6 ← n
     | Out
```

Fig.17 Mathcad program

Example 8

After six iterations, the results are

$$[VLL_{abc}] = \begin{bmatrix} 232.9\angle 28.3 \\ 231.0\angle -91.4 \\ 233.1\angle 148.9 \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 540.3\angle -19.5 \\ 593.6\angle -161.5 \\ 372.8\angle 81.7 \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 10.4\angle -19.5 \\ 11.4\angle -161.5 \\ 7.2\angle 81.7 \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} 429.3\angle 2.4 \\ 216.5\angle -128.3 \\ 214.5\angle 112.0 \end{bmatrix}$$

$$V_{unbalance} = 0.59\%$$

This example demonstrates that a small change in the Mathcad program can be made to represent the delta–delta transformer connection.

Open Delta-Open Delta

The open delta–open delta transformer connection can be connected in three different ways. Fig.18 shows the connection using phase *AB* and *BC*.

The relationship between the primary line-to-line voltages and the secondary ideal voltage is given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \quad (151)$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

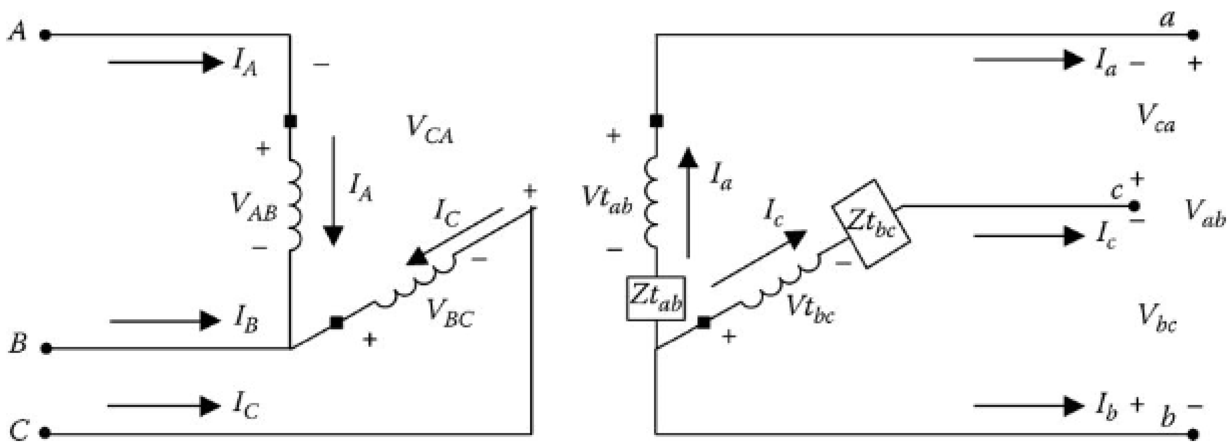


Fig.18 Open delta-open delta using phases AB and BC

Open Delta-Open Delta

The last row of the matrix $[AV]$ is the result that the sum of the line-to-line voltages must be equal to zero.

The relationship between the secondary and primary line currents is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \qquad (152)$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \qquad [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

The primary line currents as a function of the secondary line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (153)$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \qquad [d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Open Delta-Open Delta

The ideal secondary voltages are given by

$$Vt_{ab} = V_{ab} + Zt_{ab} \cdot I_a \qquad (154)$$

$$Vt_{bc} = V_{bc} + Zt_{bc} \cdot I_c$$

The primary line-to-line voltages as a function of the secondary line-to-line voltages are given by

$$V_{AB} = n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \qquad (155)$$

$$V_{BC} = n_t \cdot Vt_{bc} = n_t \cdot V_{bc} + n_t \cdot Zt_{bc} \cdot I_c$$

The sum of the primary line-to-line voltages must equal zero. Therefore, the voltage V_{CA} is given by

$$V_{AB} = -(V_{AB} + V_{AB}) = -n_t \cdot (V_{ab} + n_t \cdot Zt_{ab} + V_{bc} + n_t \cdot Zt_{bc}) \qquad (156)$$

$$V_{CA} = -n_t \cdot V_{ab} - n_t \cdot Zt_{ab} - n_t \cdot V_{bc} - n_t \cdot Zt_{bc})$$

Open Delta-Open Delta

$$V_{AB} = n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \quad (155)$$

$$V_{BC} = n_t \cdot Vt_{bc} = n_t \cdot V_{bc} + n_t \cdot Zt_{bc} \cdot I_c$$

$$V_{AB} = -(V_{AB} + V_{AB}) = -n_t \cdot (V_{ab} + n_t \cdot Zt_{ab} + V_{bc} + n_t \cdot Zt_{bc}) \quad (156)$$

$$V_{CA} = -n_t \cdot V_{ab} - n_t \cdot Zt_{ab} - n_t \cdot V_{bc} - n_t \cdot Zt_{bc})$$

Equations (155) and (156) can be put into matrix form to create the backward sweep voltage equation:

$$[VLL_{ABC}] = [x] \cdot [VLL_{abc}] + [BI] \cdot [I_{abc}] \quad (157)$$

$$[x] = [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

$$[BI] = n_t \cdot \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix}$$

Open Delta-Open Delta

$$[VLL_{ABC}] = [x] \cdot [VLL_{abc}] + [BI] \cdot [I_{abc}] \tag{157}$$

Equation (157) gives the backward sweep equation in terms of line-to-line voltages. In order to convert the equation to equivalent line-to-neutral voltages, the $[W]$ and $[D]$ matrices are applied to Equation (157):

$$[VLL_{ABC}] = [AV] \cdot [VLL_{abc}] + [y] \cdot [I_{abc}] \tag{158}$$

$$[VLN_{ABC}] = [W] \cdot [VLL_{abc}] = [W] \cdot [AV] \cdot [D] \cdot [VLN_{abc}] + [W] \cdot [BI] \cdot [I_{abc}]$$

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$

$$[a_t] = [W] \cdot [AV] \cdot [D]$$

$$[b_t] = [W] \cdot [BI]$$

Open Delta-Open Delta

The forward sweep equation can be derived by defining the ideal voltages as a function of the primary line-to-line voltages:

$$\begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} \qquad (159.a)$$

$$[Vt_{abc}] = [BV] \cdot [VLL_{ABC}] \qquad [BV] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

The ideal secondary voltages as a function of the terminal line-to-line voltages are given by

$$\begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (159.b)$$

$$[Vt_{abc}] = [VLL_{abc}] + [BI] \cdot [I_{abc}]$$

$$[BI] = \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix}$$

Open Delta-Open Delta

$$[Vt_{abc}] = [BV] \cdot [VLL_{ABC}] \quad (159.a)$$

$$[Vt_{abc}] = [VLL_{abc}] + [BI] \cdot [I_{abc}] \quad (159.b)$$

$$[VLL_{ABC}] = [AV] \cdot [VLL_{abc}] + [y] \cdot [I_{abc}] \quad (158)$$

Equate Equation (158) to Equation (159):

$$[BV] \cdot [VLL_{ABC}] = [VLL_{abc}] + [BI] \cdot [I_{abc}]$$

$$[VLL_{abc}] = [BV] \cdot [VLL_{ABC}] - [BI] \cdot [I_{abc}] \quad (160)$$

Equation (160) gives the forward sweep equation in terms of line-to-line voltages. As before, the $[W]$ and $[D]$ matrices are used to convert Equation (160) using line-to-neutral voltages:

$$[VLL_{abc}] = [BV] \cdot [VLL_{ABC}] - [BI] \cdot [I_{abc}] \quad (161)$$

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] = [W] \cdot [BV] \cdot [D] \cdot [VLN_{ABC}] - [W] \cdot [BI] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}]$$

$$[A_t] = [W] \cdot [BV] \cdot [D]$$

$$[B_t] = [W] \cdot [BI]$$

Thank You!